The Black Scholes Formula

What Are Options?

Options are contracts that allow one to buy or sell a security or assets a set price. Their are two main classes of options: calls and puts. A call allows the person who buys the contract to buy the security or asset, and a put allows the person who buys to contract to sell the security or asset. So for example, if I buy a put on Apple shares with a given "strike price," I've bought the contractual right to sell those shares at that given price.

Their are many types of options contracts, but the two most common are called European Options and American Options. If I've bought a European option, I can only buy or sell the given security or asset on the maturity date. So for example, if I've bought a European put on Apple shares with a strike price of $200 and a maturity date of May 2nd, I can only exercise the right to sell the shares on May 2nd. If I had bought an American put on Apple shares with a strike price of $200 and a maturity date of May 2nd, I could exercise the right to sell the share at $200 any time before May 2nd.

What is the Black-Scholes model?

The Black-Scholes model is the most popular model for the evaluation of an option contract's value. It it specifically designed to value European options. Because American options can be exercised at any time before the maturity date, the evaluation of their value is much more complicated. To value American options, people generally use the Bjerksund-Stensland model. However, because the Bjerksund-Stensland model is so much more complex, the Black-Shcoles model is can also used to provide a lose approximation of American option's values.

The Formula

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

C = Call Option Price

N = Cumulative Distribution Function of the Normal Normal Distribution [F(x) = P(X<x) where x is a specified value and X is random]

S_t = Spot Price of the Asset

K = Strike Price

r = Risk-Free Interest Rate

Robert Merton later improved the formula by replacing r with r-δ where r is risk free interest rate and δ is the dividend yield of the asset.

t = Time to Maturity

σ = Volatility of the Asset

Initialization Code

```
In[ ]:= d1[S_, k_, r_, δ_, σ_, T_] = (Log[S / k] + (r - δ + σ^2 / 2) T) / (σ √T);

In[ ]:= d2[S_, k_, r_, δ_, σ_, T_] = (Log[S / k] + (r - δ - σ^2 / 2) T) / (σ √T);

In[ ]:= 𝒩[z_] = (1 + Erf[z / √2]) / 2;

In[ ]:= BSCall[S_, k_, r_, δ_, σ_, T_] :=
          S ⅇ^(-δ T) 𝒩[d1[S, k, r, δ, σ, T]] - k ⅇ^(-r T) 𝒩[d2[S, k, r, δ, σ, T]];

In[ ]:= BSPut[S_, k_, r_, δ_, σ_, T_] :=
          k ⅇ^(-r T) 𝒩[-d2[S, k, r, δ, σ, T]] - S ⅇ^(-δ T) 𝒩[-d1[S, k, r, δ, σ, T]];
```

Manipulate

In[ ]:=
```
Manipulate[d1;
  d2;
  𝒩;
  BSCall;
  BSPut;
  Plot[BSOption[S, annotation, r, δ, σ, t], {S, 70., 140.},
   AxesLabel → {"stock price", "option price"}, ImageSize → 400,
   ImagePadding → {{25, 55}, {25, 25}}, AxesOrigin → {Automatic, 0}],
  {{BSOption, BSCall, "option"},
   {BSCall → "call", BSPut → "put"}},
  {{annotation, 100., "strike price"}, 50., 150., Appearance → "Labeled"},
   {{r, .05, "interest rate"}, .02, .1, Appearance → "Labeled"},
  {{δ, .004, "dividend yield"}, .002, .05, Appearance → "Labeled"},
  {{σ, .3, "volatility"}, .2, .5, Appearance → "Labeled"},
  {{t, .1, "time to maturity"}, .001, 1., Appearance → "Labeled"},
   SaveDefinitions → True, AutorunSequencing → {1, 2, 5, 6}]
```

Out[ ]=