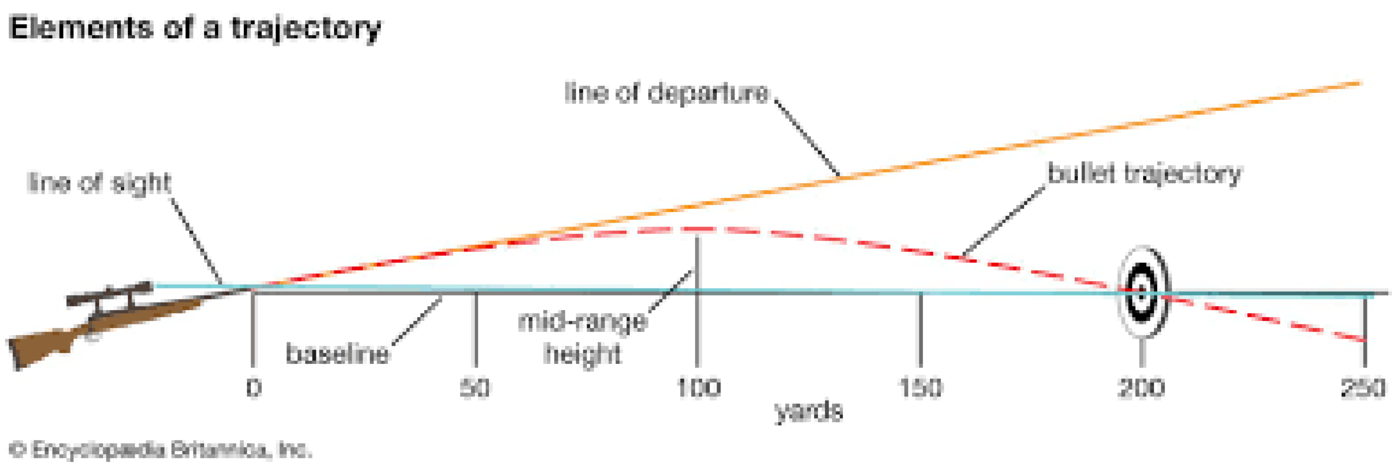
External Rifle Ballistics

Introduction to Ballistics:

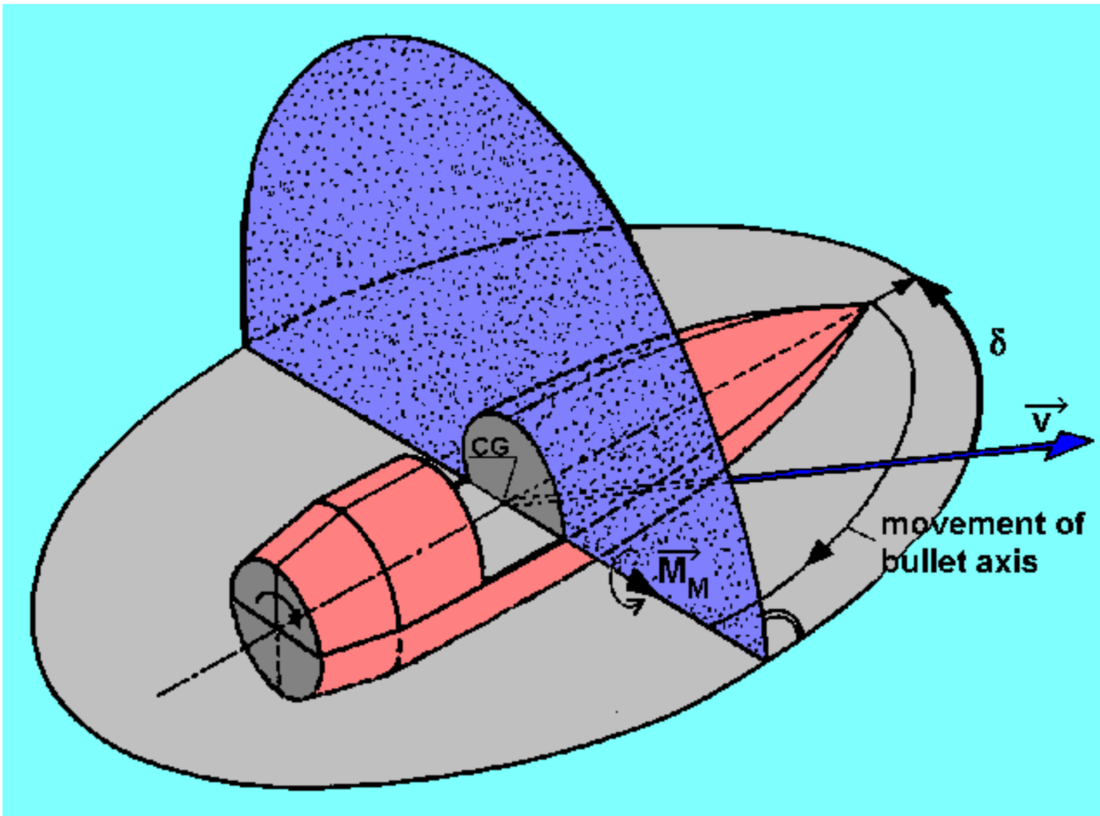
Their are two types of ballistics analysis, external and internal ballistic analysis. The two types of ballistic analysis provide separate forms of data. Internal ballistics revolved around gun mechanics and is useful to gun designers who are attempts to alter the mechanics of a rifle. External ballistics are important to marksmen who care less about the gun's mechanics and more about the bullets trajection. This notebook attempts to simulate the external ballistics of a bullet, showing where the bullet will hit in 100 yards.



Forces Acting on a Bullet:

The main forces acting on a Bullet are: gravity, drag, and wind. For longer ranges there are more meso variables. Such as the Magnus effect or possion which is where the spin of a bullet creates an imbalance in air pressure pushing it up. These affects however are pretty miniscule in the short term and only become ore important for longer range guns. The more advanced model to calculate the meso variables is called the 6 degrees of freedom model. This model accounts for all the movements of the bullet but requires a large amount of data to complete. What we have is a model that uses three directions to show where the bullet is headed.

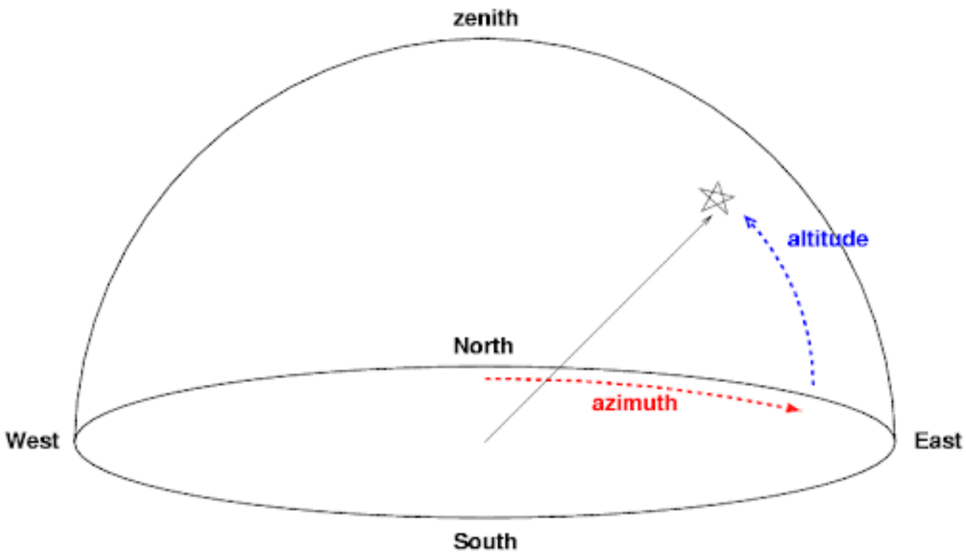
This model will also used simplified versions of drag and wind resistance. Their are multiple equations that mock up these variables. For the drag force, the Pejsa model is a more advanced model to calculate Drag, this model calculated the drag effect using a pre-determined coefficient. Additionally, the effect of gravity on a bullet drop is typically modeled through a bullet drop diagram. This diagram shows how the force of gravity will alter the bullets trajectory.



Coordinates of the Bullet:

This notebook calculates the bullet's trajectory in three directions. If our downrange direction is x--and we think of that as North--our *westward* direction is y, and our vertical direction is z.

We will identify θ as the angle called "altitude."
We will identify ϕ as the angle called "azimuth."



Initial Conditions:

These are the variables that the equation uses to calculate the bullet's trajectoy.

t is time,
x and z are the horizontal and vertical dimensions,
V_x and V_z represent the horizontal and vertical velocities,
V_y represents the lateral velocity,
θ is the initial rifle bore angle,
C_d stands for the overall drag coefficient due to air resistance,
g is the force of gravity, and
h is the height of the telescopic sight above the rifle bore.
w is the wind speed

$x(0) = 0$,
$y(0)=0$,
$z(0) = -h$.

Equations:

This demonstration shows the trajectory of a rifle bullet determined as a function of bullet characteristics and settings of the telescopic sight of the rifle.These equations are solved with `NDSolveValue`. The result shows the bullet trajectory and bullet impact in a target at 100 yards for different settings of the telescopic sight. Published bullet velocities at the muzzle and at 100 yards are used to determine the overall drag coefficient C_d. The equations describing the bullet trajectory are:

These derivatives find the velocity of the bullet in the three different directions:
$dx/dt = V_x$,
$dy/dt = V_z$,
$dy/dt = V_y$,

This is the wind force. Wx is calculated through the cos of ϕ (the wind force). Wx is calculated through the sin of ϕ (the wind force):
$w_x = w\cos\phi_w$
$w_y = -w\sin\phi_w$

These are the simplified versions of the net force diagram:
$u_x = v_x + w_x$
$u_y = v_y + w_y$
$u_z = v_z$

This is the applied version that calculated the force of the bullet with the contextualized factors of the bullet:
$u_x = \sqrt{V_x^2 + V_z^2 + V_y^2}\cos\theta\cos\phi + W_x$
$u_y = \sqrt{V_x^2 + V_z^2 + V_y^2}\cos\theta\sin\phi + W_y$
$u_z = \sqrt{V_x^2 + V_z^2 + V_y^2}\ \sin\theta$

This is what we used:
$dV_x/dt = -C_\mathrm{d}\,u_x\ \sqrt{u_x^2 + u_z^2 + u_y^2}$,
$dV_y/dt = -C_\mathrm{d}\,u_y\ \sqrt{u_x^2 + u_z^2 + u_y^2} - \mathrm{wd}$,
$dV_z/dt = -C_\mathrm{d}\,u_z\ \sqrt{u_x^2 + u_z^2 + u_y^2} - g$,

Code:

```
In[5]:= F[Cd_, θ_, ϕ_, h_, v0_, wD_] := Quiet@ ({X, Y, Z, Vx, Vy, Vz} =
      Quiet@NDSolveValue[{x''[t] == -Cd x'[t] Sqrt[x'[t]^2 + z'[ t]^2 + y'[t]^2],
         z''[t] == -Cd z'[t] Sqrt[x'[t]^2 + z'[t]^2 y'[t]^2] - g,
         y''[t] == -Cd y'[t] Sqrt[x'[t]^2 + z'[ t]^2 + y'[t]^2] - wD,
         x[0] == 0, z[0] == -h / 12, y[0] == 0, x'[0 ] == v0 Cos[θ Degree] Cos[ϕ Degree],
         z'[0] == v0 Sin[θ Degree], y'[0] == v0 Cos[θ Degree] Sin[ϕ Degree],
         WhenEvent[x[t] == 300, t100 = t],
         WhenEvent[z[t] == 300, yt100 = t],
         WhenEvent[z[t] > 0, tzeroUp = t],
         WhenEvent[z[t] < 0, tzero = t],
         WhenEvent[x[t] == 4000, {tend = t, "StopIntegration"}]}, {x, y, z, x', y',
         z'}, {t, 0, Infinity}]; Sqrt[Vx[t100]^2 + Vz[t100]^2 + Vy[t100]^2] );


Manipulate[Module[{G, κ},
   Clear[tzero];
   G = Quiet@FunctionInterpolation[F[Cd, θ, ϕ, h, v0, wD], {Cd, 0, 0.0008}];
   κ = Quiet@NSolve[(G[Cd] - f / 100 v0) == 0, Cd][[-1, 1, 2]];
   F[κ, θ, ϕ, h, v0, wD];
   Column[{
     ParametricPlot[Evaluate[Table[{i Cos[u], i Sin[u]}, {i, 0, 30, 5}]], {u, 0, 2 Pi},
      AxesStyle → Black, ImageSize → 1.125 {280, 200},
      PlotLabel → Column[{"point of aim is zero in target at 100 yards",
         If[NumericQ[tzero],
          Row[{"bullet impacts zero at ", Style[NumberForm[X[tzeroUp] / 3, 2], Red],
            " and ", Style[NumberForm[X[tzero] / 3, 4], Red], " yards"}],
          Style["increase initial angle or select faster bullet", Red]]}],
      ImagePadding → {{Automatic, Automatic}, {Automatic, Automatic}},
      Epilog → {
        Text[
         Row[{NumberForm[12 Z[t100], 2], " inches"}], {20, 8}, Background → White],
        Text["at 100 yards", {21, 3.5}, Background → White],
        {Red, PointSize[0.05], Point[{{12 Y[t100], 12 Z[t100 ]}}]}}]}]],
  Grid[{
    {Column[{Style["bullet properties", Bold]}],
     Column[{Style["scope setting", Bold]}]},
    {Control@{{v0, 3000., "muzzle velocity (feet/second)"},
       2000., 3500., 1, ImageSize → Tiny, Appearance → "Labeled"},
     Control@{{h, 1.0, "scope height (inches)"},
       1., 2.5, 0.25, ImageSize → Tiny, Appearance → "Labeled"}},
```

```
  {Control@{{f, 93., "% muzzle velocity at 100 yards"},
     80., 97., 1, ImageSize → Tiny, Appearance → "Labeled"},
   Control@{{θ, 0.17, "initial altitude (degrees)"},
     0., 0.31, 0.0010, ImageSize → Tiny, Appearance → "Labeled"},
   Control@{{ϕ, 0.17, "initial azimuth (degrees)"}, 0., 0.31,
     0.0010, ImageSize → Tiny, Appearance → "Labeled"},
   Control@{{wD, 20, "wind drag"},
     -20, 20, 0.0010, ImageSize → Tiny, Appearance → "Labeled"}
  }}],
TrackedSymbols :→ {v0, h, f , θ, ϕ , wD},
ControlPlacement → Top,
SaveDefinitions → True,
AutorunSequencing → {1, 4}, Alignment → Center]
```

Out[6]=