Quantum Physics, Preparation for Friday, Feb. 9

Study the Rest of Q6 from *Six Ideas*

Do as much review of torque and angular momentum as you have to to understand the Stern-Gerlach apparatus.

Continue Studying Churchill, Brown, and Verhey (CBV)

You have already studied Sections 1-4 of CBV. Continue into Sections 5 and 6. That will be enough complex variables to get you going in quantum mechanics. As interesting as Sections 7 and 8 are, we won't need them.

For Problem Set 7

Complex variables, powers, exponentials, and derivatives

1. In the last problem on Problem Set 6, you found that

$(\cos\ \theta\ +\ i\ \sin\theta)^3 = \cos 3\theta + i\ \sin 3\ \theta$

You used trig identities to do it. Now we will stand this problem on its head. We will derive trig identities from the general formula:

$(\cos\ \theta\ +\ i\ \sin\theta)^n = \cos\ n\ \theta + i\ \sin\ n\ \theta$

(a) Write down the equation above for $n = 2$ and FOIL the square into four terms, two of which are the same. Now it must be true that the real part of the LHS equals the real part of the RHS. It must also be true that the imaginary part of the LHS equals the imaginary part of the RHS. Use those two facts to write down trig identities for $\cos 2\ \theta$ and $\sin 2\ \theta$.

(b) Write down the equation above for $n = 4$. Expand the LHS into 16 terms, a bunch of which are the same, but save yourself a bunch of time, by using using the fourth row of Pascal's triangle, which is 1 4 6 4 1. As in part (a) use the real part of the resulting equation and the imaginary part to get trig identities for $\cos 4\ \theta$ and $\sin 4\ \theta$.

(CONT'D on reverse)

2. Problem 2 on p. 17 of CBV

3. Assuming i, p, ω, E, and $\hbar$ are constants and that the usual rules for taking derivatives of exponentials remain true for complex exponentials, what are the following derivatives:

(a) $i\hbar \frac{d}{dt} e^{-i E t/\hbar}$

(b) $-i\hbar \frac{d}{dx} e^{i p x/\hbar}$

(c) $-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} e^{i p x/\hbar}$

(d) $i\hbar \frac{\partial}{\partial t} \left(e^{-iEt/\hbar} e^{i p x/\hbar}\right) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \left(e^{-iEt/\hbar} e^{i p x/\hbar}\right)$

In the last equation I switched from d to ∂ because both t and x are present in the function you are differentiating, but the derivatives work exactly the same way as they did in parts (a) and (c).

(e) Assuming $e^{-iEt/\hbar} e^{i p x/\hbar}$ is some kind of wave function, and (d) is some kind of wave equation, what must the relationship between E and p be for the equation to be satisfied?

Quantum Mechanics With the Stern-Gerlach Apparatus

4. Q6B.2, p. 99
5. Q6B.4, p. 99
6. Q6M.4, p. 99 classical precession, review Section Q6.3 before attempting this problem