Quantum Physics, Preparation for Friday, Feb. 16

Study Q9.1 to Q9.3 of *Six Ideas*

Q9.1 and Q9.2 are where the new ideas about wavefunctions are introduced. These are the two sections I was introducing at the end of Tuesday's class.

Q9.3 is an accurate and to-the-point version of the issue of "The Collapse of the Wavefunction."

Q9.4 and Q9.5 you can wait to study until the beginning of next term, although the third problem below is designed to help you start thinking about Section Q9.4, "The Heisenberg Uncertainty Principle." In short, this principle says that if the position is very accurately known, then the momentum is not, and if the momentum is very accurately unknown, the position is not. The principle is captured in Equation Q9.17.

For Problem Set 9

Basic and Intermediate Problems

1. Q9B.6, p. 147

For this problem, also find the value of A that properly normalizes this wavefunction (although Moore did not ask for it). HINTS: The average value of the $\sin^2$ function over any complete number of half-periods is 1/2, and the answer is not just $A = \sqrt{2}$.

2. Q9M.2, p. 147

3. Q9M.5, p. 148, this is your first encounter with the Gaussian (actually, your second encounter, because I put this integral on the board on Tuesday). HINT: Moore is suggesting that you exploit the fact that 0.1nm is a lot less than 1.5nm. It may help you to make a good plot of the Gaussian.

Challenging Multi-Part Problems

4. Q7D.2, p. 148 (a), (b), and (c) only.

5. The last problem is on the reverse.

It introduces you to the "c" in Dirac's <bra|c|ket> notation.

5. Many measurements are not just yes or no. They have values. A super-important measurement for the electron is the value of its spin. You have been told that the |+z> state has spin $\hbar/2$ in the +z direction and the |-z> state has spin $\hbar/2$ in the -z direction. We capture this idea in something called an operator, and in equations, the S_z operator satisfies:

$S_z \mid +z> = \frac{\hbar}{2} \mid +z>$ and $S_z \mid -z> = \frac{-\hbar}{2} \mid -z>$.

There are lots of other equations you know too then, for example:

$S_x \mid +x> = \frac{\hbar}{2} \mid +x>$ and $S_x \mid -x> = \frac{-\hbar}{2} \mid -x>$.

The rule is, if you want to know ***the expectation value*** of an operator C in the state $\mid \psi>$, then you calculate $<\psi \mid C \mid \psi>$. It is very interesting to know other things about the operator C, such as $<\phi \mid C \mid \psi>$. In a system in which there are really only two states, and all other states are linear combinations of the two states, C can be thought of as a 2x2 matrix. The matrix entries (more formally called ***matrix elements***) depend on what states you choose as your basis states.

(a) In the |+z>, |-z> basis, the operator S_z can be thought of as the following 2x2 matrix:

$$S_z = \begin{pmatrix} <+z \mid S_z \mid +z> & <+z \mid S_z \mid -z> \\ <-z \mid S_z \mid +z> & <-z \mid S_z \mid -z> \end{pmatrix}$$

You now know enough to compute this matrix. Because every entry will have $\hbar/2$ in it, let’s write it as $S_z = \frac{\hbar}{2}\sigma_z$. What is the 2x2 matrix σ_z?

(b) Now we will calculate the matrix for S_x in the z basis. A sticking point is, you don’t know any of the needed entries, each of which is of the form $<\pm z \mid S_x \mid \pm z>$. But that is ok! Go back to Table Q7.1. In terms of states, that table says $\mid +x> = \frac{1}{\sqrt{2}}(\mid +z> + \mid -z>)$ and $\mid -x> = \frac{1}{\sqrt{2}}(\mid +z> - \mid -z>)$. For part (b), I just want you to solve those equations for $\mid +z>$ and $\mid -z>$ in terms of $\mid +x>$ and $\mid -x>$.

(c) Now that you have expressions for $\mid +z>$ and $\mid -z>$ in terms of $\mid +x>$ and $\mid -x>$, put those expressions into the matrix for S_x and simplify. Start with:

$$S_x = \begin{pmatrix} <+z \mid S_x \mid +z> & <+z \mid S_x \mid -z> \\ <-z \mid S_x \mid +z> & <-z \mid S_x \mid -z> \end{pmatrix}$$

As with S_z, every entry of S_x will have $\hbar/2$ in it, so let’s write it as $S_x = \frac{\hbar}{2}\sigma_x$. What is the 2x2 matrix σ_x?

EPILOG: To complete the story of S_x, S_y, and S_z, I’ll just tell you that $S_y = \frac{\hbar}{2}\sigma_y$ and $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.