Quantum Physics, Preparation for Friday, Apr. 26

Reading

Rebecca wanted to make as much progress on relativity as we can in just the 10 minutes of last Friday's class, plus the time we have this Friday. So I am going to try to get you into one of the oddest things in relativity, which is that time is no longer absolute. Not only does it arguably run at different rates when observers compare notebooks, but also, one observer can say two things happened simultaneously, and another observer will not even agree to that!

Do the problem set first, and then read Chapter IX of *Relativity* by Albert Einstein. The book was published in 1916 in German and translated to English in 1920 by Robert Lawson. *Relativity* is Einstein's attempt to make his 1905 theory widely understood. There are no oversimplifications in Einstein's explanation of special relativity. Only better, simpler explanations than he originally used. Successive generations keep refining the arguments. If you want to read more, read anything by John Archibald Wheeler as a (co-)author. Wheeler was Richard Feynman's and Kip Thorne's thesis advisor.

To help you keep the big picture clear, let me say at the outset that the three subjects I am going to introduce you to are:

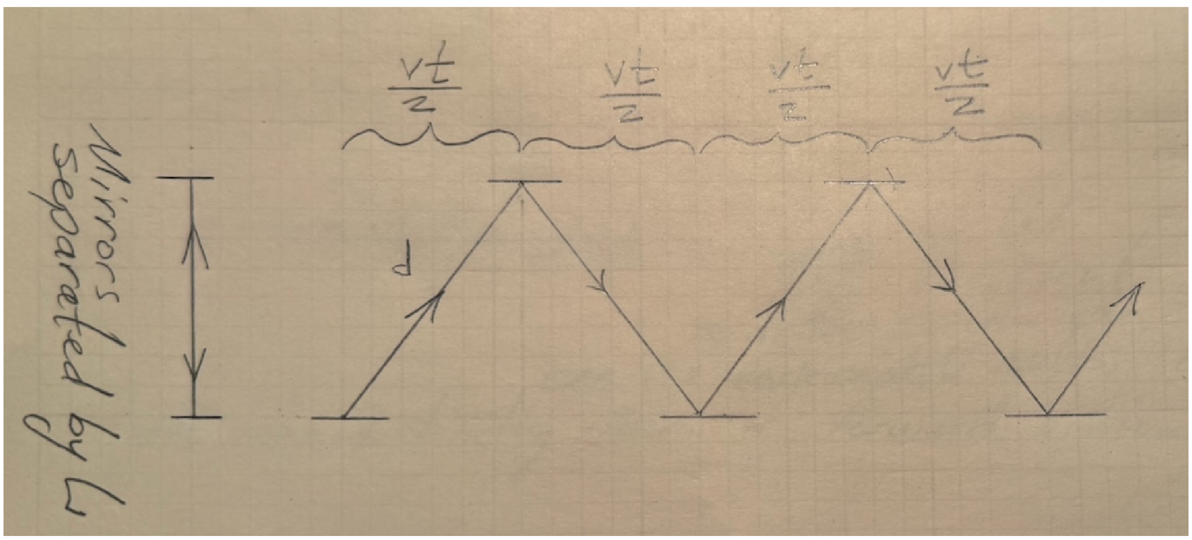
1. Moving clocks run slow (too long between ticks!)
2. Moving meter sticks are shortened (but only in the direction of their motion!)
3. Simultaneity is relative

All of these disorienting facts are consequences of the benign-looking statement, "the speed of light is constant."

For Problem Set 20

Moving Clocks Run Slow

Consider the situation below, as viewed by two observers. In the left digram, according to the observer holding the two mirrors, a photon simply bounces back and forth. But this observer is coasting by on a very high-speed a rocket! We are going to watch this from our space station as the rocket coasts by at $\frac{3}{5}$ of the speed of light. It is important that the mirrors are held perpendicular to the direction of motion of the rocket ship going past the space station. [Recall the paintbrush argument from last Friday.] The space station observer sees the photon bouncing back and forth as shown in the right diagram.



1. Let τ be how long a photon takes for a round trip, according to the observer holding the two mirrors. Write down an expression for L in terms of τ and c.

2. Write down a relationship between d (the hypotenuse), $\frac{v\,t}{2}$ (the base), and L (the height) of the triangle defined in the right diagram. You just use the Pythagorean theorem for this.

3. In what you just did, v is the rocket ship's speed according to the observer on the space station, and t is how long a round trip of the photon takes according to the observer on the space station. What is another relationship that you can write down between d, t, and c just using "the speed of light is constant." In other words, $2\,d$ is traversed by the photon in time t according to the space station observer, and c is the speed of light. That implies a pretty simple relationship.

4. Solve for d in the equation you got in Problem 2. Solve for d again in the equation you got in Problem 3. Now you have two expressions for d. Set them equal to each other.

5. In Problem 1 you solved for L. Use that to get rid of L in the equation you got in Problem 4. This step has most of the algebra in the "moving clocks run slow" argument. Just rearrange until you have an equation for t in terms of τ. Probably the first thing you'll do is cancel all the 2's. Then square both sides of the equation. After that it should be straightforward to solve for t in terms of τ.

CROSS-CHECK: L and d are gone. This expression will just involve v and c. Frankly, everything that follows on this problem set is going to be a mess if you don't get this right, so I will tell you that the answer is

$$t = \frac{\tau}{\sqrt{1-v^2/c^2}}$$

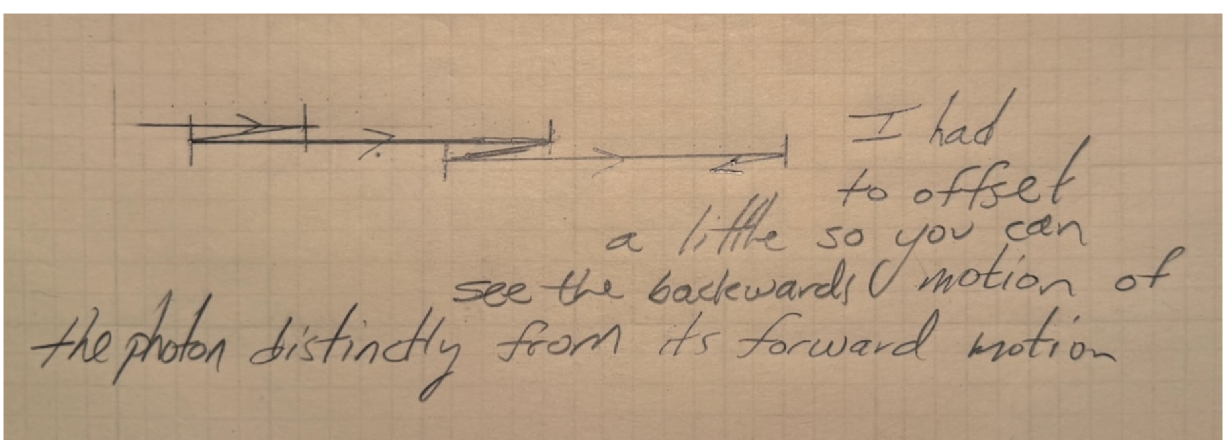
Go back and find legitimate mistakes and correct them until you get the right answer.

6. If you plug in $v = \frac{3}{5}c$ into what you got in 5, by what factor does the rocket ship's clock run slow according to the space station observer? So if someone's life was measured to be 80 years long in the rocket frame, and they aged more slowly by this factor, how long would they be observed to live by the space station observer?

Meter Sticks are Foreshortened

7. The person on the rocket ship re-orients their two mirrors to be in the direction of their motion past the space station. Obviously this does not change the ticking speed according to them, nor can it change the ticking speed according to the person on the space station. "Obviously" is a big word. Give an argument. I am not going to grade this argument, but I want you to share whatever argument you use to convince yourself that the orientation cannot matter with your classmates on Friday. Write your argument down in 2-3 succinct sentences.

Below is how the motion of the photon now looks from the space station:



8. Let’s assume the mirror distance is now L/γ according to the person on the space station. The paintbrush argument I gave last Friday is no longer applicable (why?), so that’s why we are allowing for a changed mirror distance. The rocket ship is still going by the space station at speed v. If it takes t_R for the photon to go from the back mirror to the front mirror the front mirror will have moved $v t_R$. So the total distance the photon goes while catching up to the front mirror is $\frac{L}{\gamma} + v\, t_R$. But the total distance is also $c\, t_R$ because the speed of light is constant. So you have $\frac{L}{\gamma} + v\, t_R = c t_R$. Solve this equation for t_R.

9. For the photon to go from the front mirror to the back mirror, we have a similar equation, $\frac{L}{\gamma} - v\ t_L = c\, t_L$, except this time the back mirror is catching up to the photon, hence the minus sign. Solve this equation for t_L.

10. By your argument in 7, $t_L + t_R$ must still be t. So add what you got in 8 and what you got in 9 together and set it equal to t. Meanwhile, what you got in 5 is still true, so replace t by $\frac{\tau}{\sqrt{1 - v^2/c^2}}$ and also replace τ by $\frac{2L}{c}$ which you got back in Problem 1.

CROSS-CHECK AND DISCUSSION: In 10, the factors of $2\,L$ should cancel away. The same funny factor that showed up in 5, $\frac{1}{\sqrt{1 - v^2/c^2}}$, has showed up again here as γ. There it was the amount that clocks run slow, and here it is the amount that meter sticks are foreshortened. Weird.

Simultaneity is Relative

Now you are ready to read Chapter IX of Einstein. If I have helped you get to the point where you can read and debate “the relativity of simultaneity,” all the while also keeping in mind that

1. Moving clocks run slow (too long between ticks)
2. Moving meter sticks are shortened (but only in the direction of their motion)

then maybe my job is done for this semester, and I should step out, because I have already poured out all the explanation I have into this problem set :P

I hope to see some of you in whatever we do this fall, and congratulations to those of you who are moving on!