Modern Introductory Physics, Part II, Exam 1

Friday, Feb. 2, 2024 — Covering *Six Ideas,* Volume Q, Chapters Q1-C5

1A. Kinetic Energy in a Standing Wave on a String (3 pts)

Suppose a string is laid out on the x-axis from $x = 0$ to $x = L$.

(a) For a standing wave, the string's displacement in the y-direction is usually written

$y(x,\ t) = A \sin kx \sin \omega t$

Take one time derivative to find $v_y(x,\ t)$.

(b) The kinetic energy in the string is just the infinite sum of the kinetic energy of each of its infinitesimal parts. If the mass per unit length is ρ, each infinitesimal bit of length dx has mass $\rho\, dx$ and the total kinetic energy is:

$\mathrm{KE} = \int_0^L \frac{1}{2} [v_y(x,\ t)]^2\, \rho\, \mathrm{dx}$

Put in what you found in (a) for $v_y(x,\ t)$, and then pull out in front of the integral all the factors that don't depend on x. When you have done this, you will have:

$$\text{lots of factors that do not depend on } x \text{ times } \int_0^L \sin^2 k\, x\, dx$$

(c) A lovely simplification is that that integral is the same no matter what mode we are considering. I'll just tell you:

$\int_0^L \sin^2 k\, x\, dx\ =\ \frac{L}{2}$

Use the lovely simplification to write your final answer for the kinetic energy.

NOTE: The kinetic energy is proportional to $\cos^2 \omega\, t$ which is time-varying, but total energy is conserved, so it must go somewhere. We'll figure that out in the next problem.

1B. Potential Energy in a Standing Wave on a String (3 pts)

In this problem we calculate the potential energy from where the kinetic energy comes and goes.

(a) If a string is stretched by s, then its potential energy is $U = s\,T$, where this T stands for the tension in the string (not the period). I'll just tell you another fact (because it requires another integral), which is that the amount that the string is stretched, s, is:

$$s = A^2 k^2 \frac{L}{4} \sin^2 \omega t$$

Use those two facts to write an expression for U.

(b) Also you know that $T = \rho\, v_{\text{phase}}^2$ and finally that $k^2 v_{\text{phase}}^2 = \omega^2$. Use those substitutions to get a new expression for U.

(c) In 1(c) you got an expression for the kinetic energy of the string. In 2(b) you got an expression for the potential energy of the string. Together they must add up to the total energy, which ought to be constant. Use the most standard trig identity of all $\left(\sin^2 \alpha + \cos^2 \alpha = 1\right)$ to find the time-independent value of:

$$E = \text{KE} + U$$

2. Superposition (3 pts + 2pts EC)

Use the NEXT PAGE AS SCRATCH PAPER to help you get your final answers for parts (a)-(c). I WILL HAND OUT A SHEET OF GRAPH PAPER to put the final answers on.

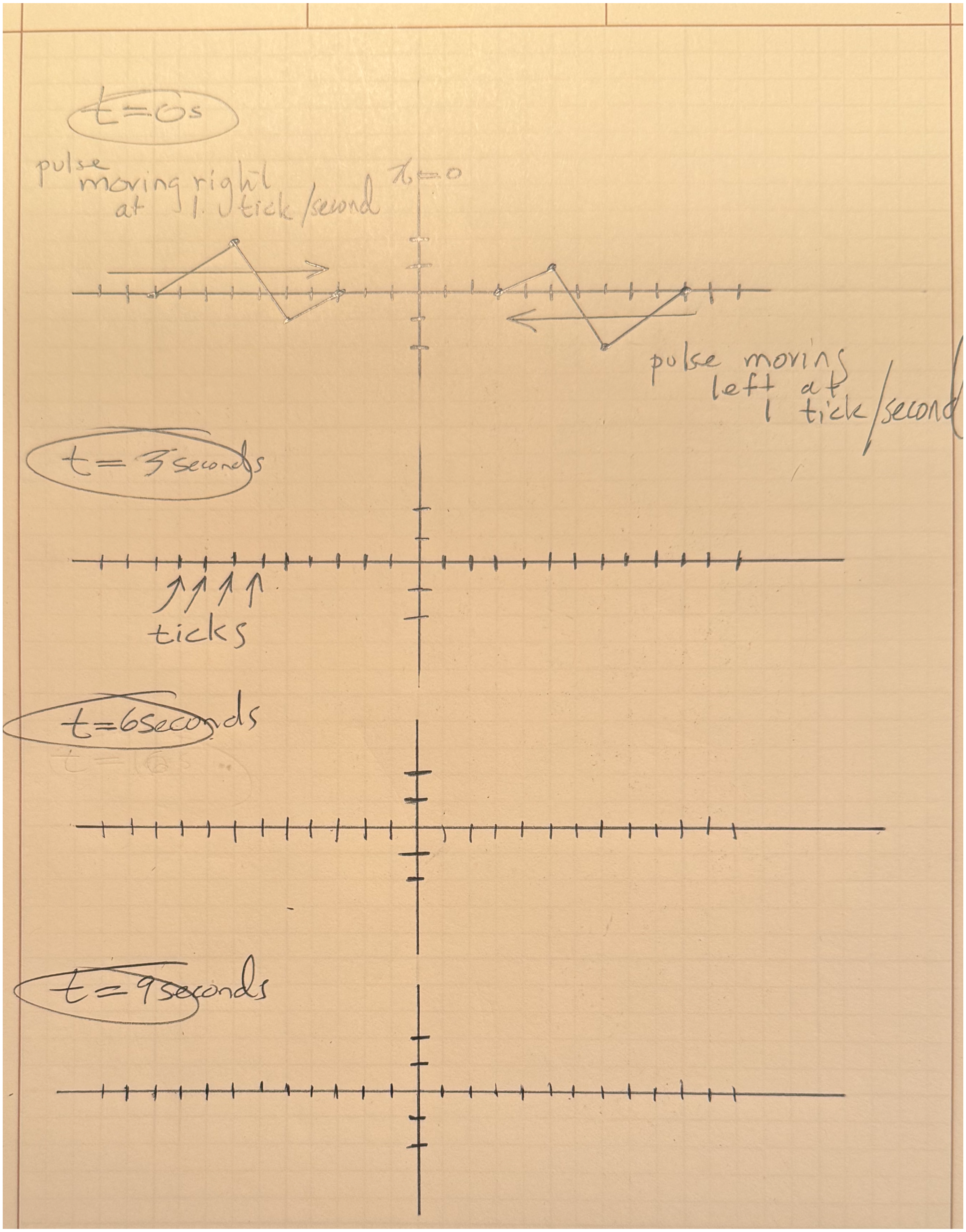
(a)-(c) Assume that the right-moving and left-moving pulses as shown at $t = 0$ seconds are each moving at the rate of 1 tick per second. Graph the superposition of the pulses at $t = 3$ seconds, $t = 6$ seconds, and $t = 9$ seconds.

HINT/CROSS-CHECK: As a cross-check on your final answer, I rigged these pulses so that at $x = 0$, their superposition is always zero. Double-check that at $x = 0$ you always have no displacement.

(d) EXTRA CREDIT (1 PT) Describe how the left half of the graph would differ if the string just ended at $x = 0$ (in other words there is no right half of the string), and instead at $x = 0$ the string was fixed?

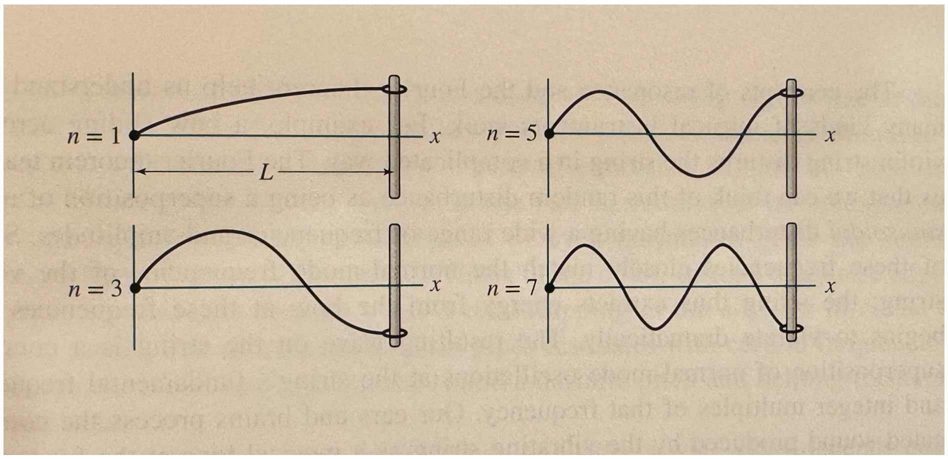
(e) MORE EXTRA CREDIT (1 PT) Describe how the left half of the graph would differ if the string just ended at $x = 0$, and instead at $x = 0$ the string was free?

SCRATCH PAPER FOR WORKING OUT PROBLEM 2



3. Superposition (4 pts)

Consider the $n = 7$ case in the diagram below:



HINT: Before answering (a), divide the wave in the n=7 case up into 7 regions each λ/4 wide. This is so everybody gets off on the right foot.

(a) What is the relationship between L and λ for the $n = 7$ case? You don't need the general formula. You are just looking at the diagram that you divided up and deducing it.

(b) Using $v_{\text{phase}} = f\,\lambda$ and the relationship you found in (a), what is f in terms of v_{phase} and L? Let's call that f_7.

(c) What is the relationship between L and λ for the $n = 1$ case?

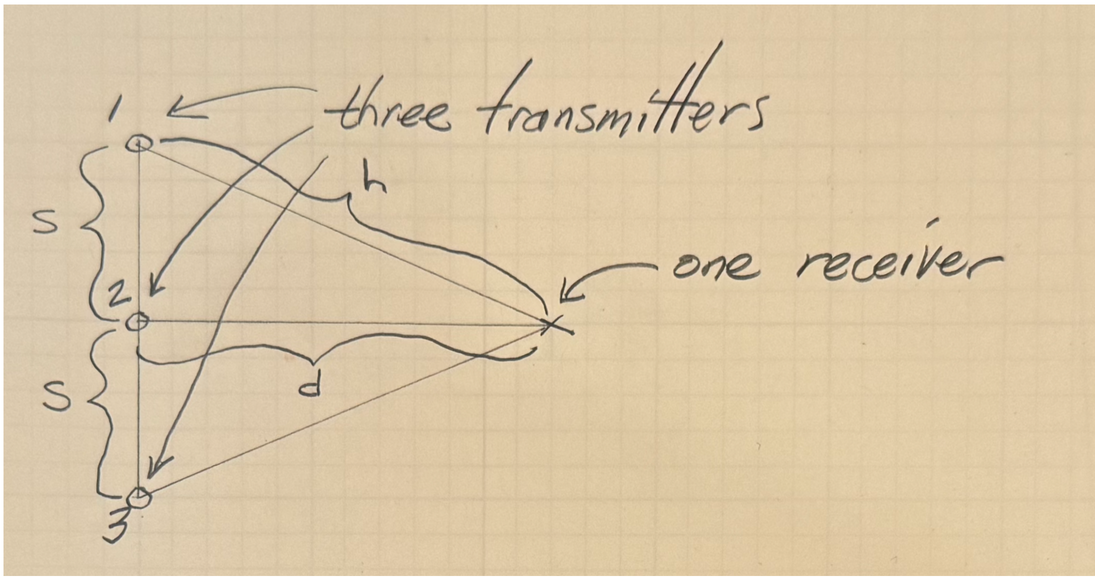
(d) Using $v_{\text{phase}} = f\,\lambda$ and the relationship you found in (c), what is f in terms of v_{phase} and L? Let's call that f_1.

(e) What is the relationship between f_7 and f_1?

4. Interference (4 pts)

We have done single-slit and double-slit to death. Here is an interference problem you haven't done. Do not use angles to solve this problem. This is a new problem type, and there aren't any angle formulas for it.

Consider three transmitters and 1 receiver as shown. I haven't drawn in the waves, but they radiate in circles from all three antennas and have wavelength λ.



(a) In terms of λ and h how many wavelengths separate transmitters 1 and 3 from the receiver?

(b) In terms of λ and d how many wavelengths separate transmitter 2 from the receiver?

(c) What is the condition that guarantees that the signals from all three transmitters constructively interfere at the receiver?

HINT: The condition will have an integer n in it.

(d) If $s = 5$ m and $d = 12$ m, what is h?

HINT: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$ is a Pythagorean triple.

(e) For the $n = 1$ case of your condition in (c) and the numbers in (d), what is λ?

PLEASE: Include units in your answers to (d) and (e) or I have to ding you.

5. Particle Nature of Light (4 pts)

Let’s say that the light from a star arrives at Earth with an intensity $I = 16{,}000$ eV per square meter per second. We are going to find out how many photons per second are collected by a telescope the size of the College’s.

(a) Use $E = h\,f$ for a photon and $c = \lambda\,f$ to get the standard formula for the energy of a photon in terms of h and λ.

(b) Use $h\,c = 1240\,\text{eV nm}$ and to make things nice and round, $\lambda = 620\,\text{nm}$ to get the number of eV per photon.

(c) The College telescope is marketed as 11”, so converting to metric and dividing by 2 its mirror has a 140mm radius. If you stick r =140mm into $\pi\,r^2$ you get a collecting area of about $\frac{1}{20}$ of a square meter.

How much power (your answer will be in eV/second) does the star rain down on a telescope of this size?

(d) Using your answers to (b) and (c) at what rate (your answer will be in photons/second) are photons collected by the telescope?

6. Wave Nature of Particles (4 pts)

(a) Combine $\lambda = h / p$ and the non-relativistic formula for kinetic energy, $K = \frac{p^2}{2m}$, to get an expression for λ in terms of K.

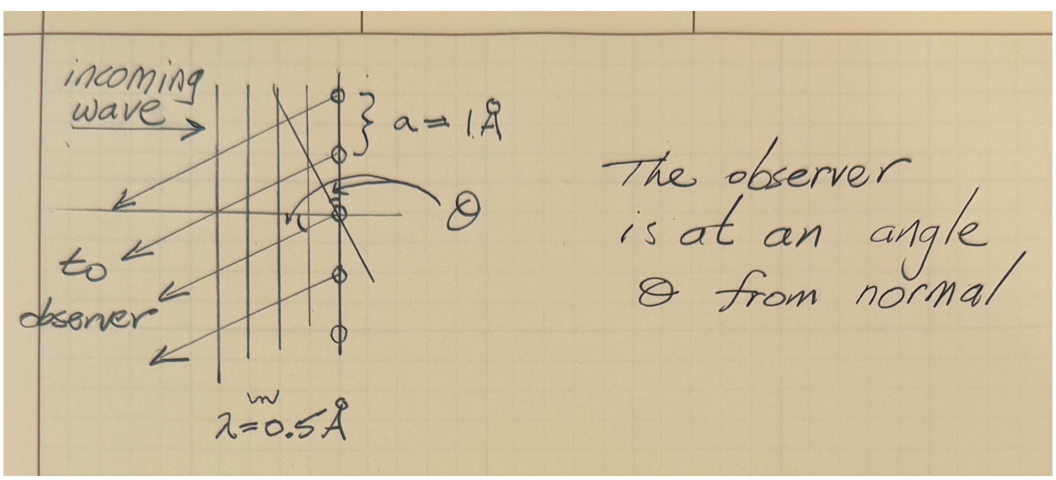
(b) It is common to rewrite the formula you got in (a) by multiplying numerator and denominator by c and also bringing the c in the denominator inside the square root where it sits nicely with the m to make $m\,c^2$. Do that.

BY THE WAY: Don't be fooled by the factors of c. This is a nonrelativistic formula with c's sprinkled around for convenience.

(c) Using $h\,c = 1240$ eV nm and for the electron $m\,c^2 = 511\,000$ eV, write down (no need to simplify) the expression for the de Broglie wavelength of an electron with $K = 3700$ eV.

NOTE: So that you don't need a calculator, I'll just tell you that your answer to (c) comes out to $\frac{1}{20}$ nm which is $\frac{1}{2}$ Å.

The answer to (c) is so small that you can't make slits and slit spacings that small. The only practical way to see the effect of the de Broglie wavelength is to scatter electrons off of crystals. In the diagram below, the circles represent the atoms in the crystal.



(d) Consider a beam of electrons with wavelength $\lambda = \frac{1}{2}$ Å coming at a crystal with spacing $a = 1$ Å. What is the condition, involving θ, a, and λ, for constructive interference? Do not make a small angle approximation.

HINT: The condition will have an integer n in it, and for the values for λ and a given, only $n = +2,\ +1,\ 0,\ -1,$ or -2 can work.

Name ____________________

1A. / 3

1B. / 3

2. / 3 (+ 2EC)

3. / 4

4. / 4

5. / 4

6. / 4

GRAND TOTAL

/ 25 MAX