# Modern Introductory Physics, Part II, Exam 4

Tuesday, April 23, 2024 — Covering Moore  Q12.4 to the end of Q15 and Problem Sets 14-18

Don't get stuck on any one problem. They are in the order that we studied them, which isn't the order of increasing difficulty. Move to the next problem if you get bogged down on one.

## 1. Numerical Methods Applied to the Drag Equation (5 pts)

Consider drag in one dimension. Because $a = \frac{dv}{dt}$, another way of writing $F = m\,a$ is $F = m\,\frac{d\,v}{d\,t}$. We can combine this with the standard formula for the drag force from last semester, $F_{\text{drag}} = -\frac{1}{2}\,C\,\rho\,A\,v^2$, and get $m\,\frac{dv}{dt} = -\frac{1}{2}\,C\,\rho\,A\,v^2$. There are too many constants in this equation to want to carry them all around in what follows, so let's just write the drag equation as $\frac{dv}{dt} = -e\,v^2$ where $e \equiv \frac{C\,\rho\,A}{2\,m}$.

(a) If you know $v(t_{i+1})$ and $v(t_i)$ and also that $t_{i+1} - t_i = \Delta\,t$, what could you write down as an approximation for $\frac{dv}{dt}$? Super-simple question. I'm just asking you to write down the usual approximation for the derivative.

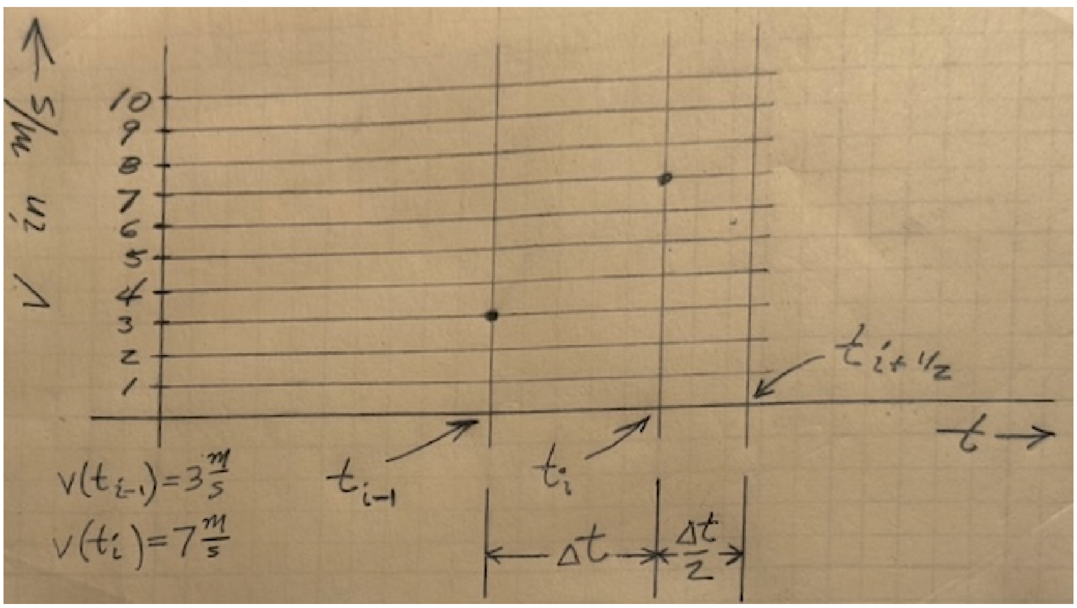
(b) Put what you wrote down in (a) into $\frac{dv}{dt} = -ev^2$ and then solve for $v(t_{i+1})$. You can just leave $-e\,v^2$ as it is. We'll get more precise about what it is in the rest of the problem.

COMMENT: To consider to help you understand part (c), *not something I want you to actually answer,* would you say that what you wrote down for $\frac{dv}{dt}$ in part (a) is a better approximation for $\frac{dv}{dt}$ at $t_i$, at $t_{i+1}$, or at the midpoint, $t_{i+1/2} \equiv \frac{t_i + t_{i+1}}{2}$? Of course in the limit that $\Delta t \to 0$ these are all three the same, but in real numerical calculations, $\Delta t$ is small but not zero, so it matters.

(c) I hope you mentally answered “midpoint” when you were considering the comment. It means that the $v$ that is in $-e\, v^2$ in part (b) would be best if it were $v(t_{i+1/2})$. So we need to figure out how to get $v(t_{i+1/2})$ from $v(t_i)$ and $v(t_{i-1})$! In this part we will do it graphically with an example.

In the graph below, $v(t_{i-1})$ is $3\ \frac{\mathrm{m}}{\mathrm{s}}$ and $v(t_i)$ is $7\ \frac{\mathrm{m}}{\mathrm{s}}$. Accurately place $v(t_{i+1/2})$ just using linear extrapolation:

$v(t_{i+1/2}) = \qquad \frac{\mathrm{m}}{\mathrm{s}}$




(d) The previous part was an illuminating graphical example. Now forget the numbers in the example (which were $3\ \frac{\mathrm{m}}{\mathrm{s}}$ and $7\ \frac{m}{\mathrm{s}}$).

Using only $v(t_i)$ and $v(t_{i-1})$, and $\Delta t \equiv t_i - t_{i-1}$, again use linear extrapolation to write down an algebraic formula for $v(t_{i+1/2})$.

HINT/HELP: Once you simplify, your answer to (d) will only contain $v(t_i)$ and $v(t_{i-1})$!

CROSSCHECK: If you plug in the numbers from the graphical example, you of course should get the exact same answer as you got graphically in (c).

(e) Stick what you got for $v(t_{i+1/2})$ in part (d) for the $v$ in $-e\,v^2$ in your answer to part (b). Now you are all done! You have $v(t_{i+1})$ in terms of $v(t_i)$ and $v(t_{i-1})$, the constant $e$ and the time $\Delta t$.

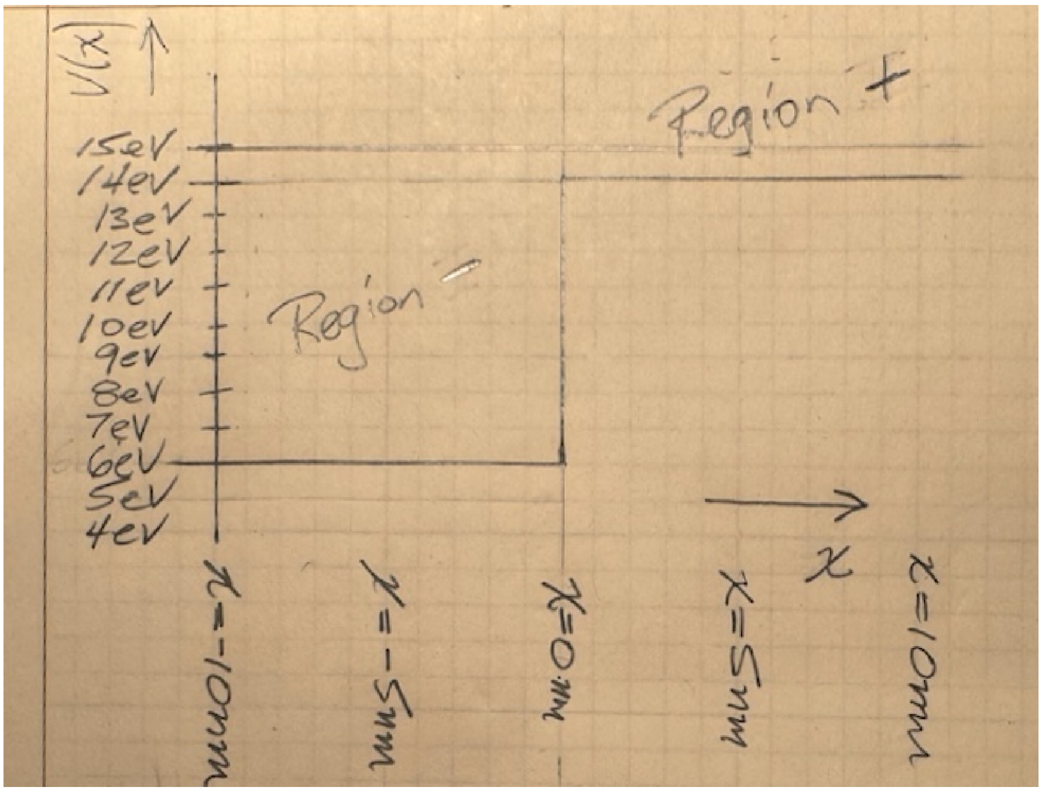
COMMENT: This is actually a good way of solving the drag equation numerically.

## 2. Qualitative Methods for Solving Schrodinger's Equation (5 pts)

At the end of Q12, you built intuition for Schrodinger's Equation using SchroSolver and by using the formulas

$$\lambda(x) = 2\pi\sqrt{\frac{-\psi(x)}{d^2\psi/dx^2}} = 2\pi\sqrt{\frac{\hbar^2}{2m(E-V(x))}} = \sqrt{\frac{h^2}{2m(E-V(x))}}$$

Below is a potential that is 6eV in Region - and 14eV in Region +.



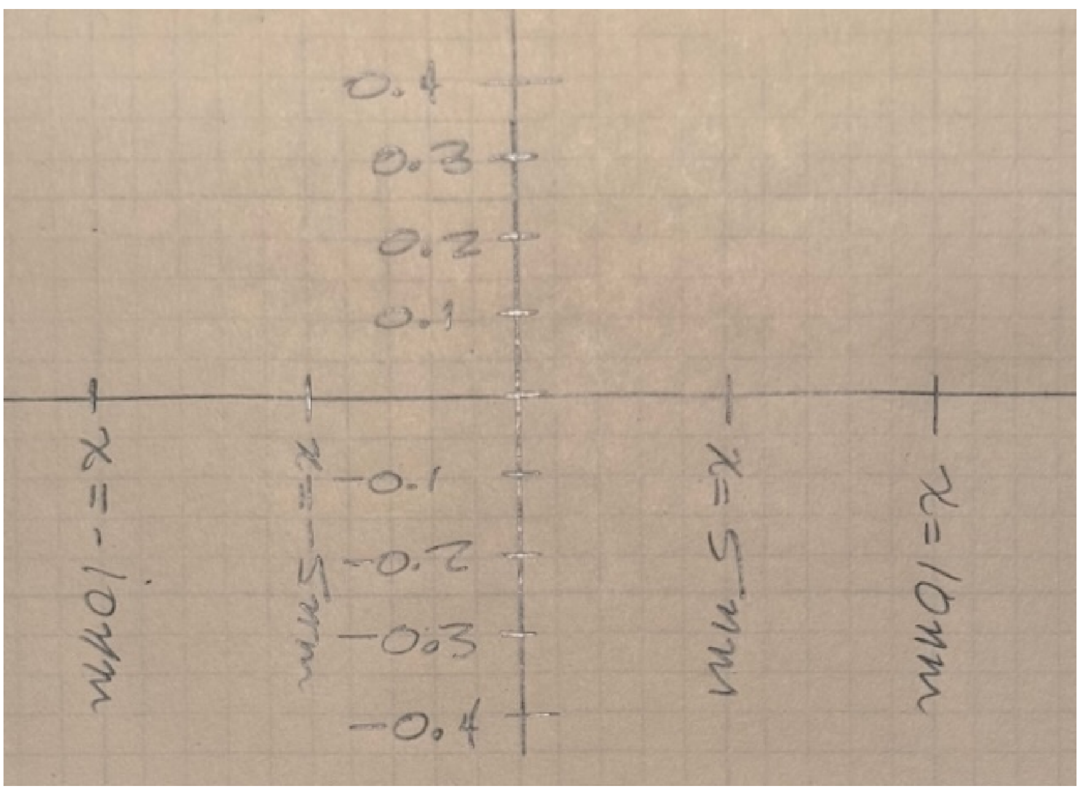

(a) If a particle has energy 15eV and if $\frac{h^2}{2m} = 36\ \text{eV}\cdot\text{nm}^2$, using $\lambda(x) = \sqrt{\frac{h^2}{2m(E-V(x))}}$, what is the wave-length $\lambda_-$ in Region - ?

(b) What is $\lambda_+$ for this particle in Region + ?

(c) If the wavefunction is $A_- \sin \frac{2\pi x}{\lambda_-}$ in Region -, what is its slope at $x = 0$? Also, if the wavefunction is $A_+ \sin \frac{2\pi x}{\lambda_+}$ in Region +, what is its slope at $x = 0$?

(d) For there to be no kink at $x = 0$ what is the relationship between $A_+$ and $A_-$? With $A_- = 0.1$ and the $\lambda_-$ and $\lambda_+$ you found above, what is $A_+$? I'm not asking you to normalize. I'm just asking you to find $A_+$ so that there is no kink.

(e) EXTRA CREDIT: Use everything you just discovered to make a nice graph of the wave function in Region - and Region +.



## 3. The “Third-Life,” $t_{1/3}$ (4 pts)

A whacky nuclear physicist decides that $t_{1/3}$ is a more convenient measure of nuclear decay rate. The whacky formula for nuclear decay rate is then:

$$N(t) = N(0)\left(\frac{1}{3}\right)^{t/t_{1/3}}$$

(a) The whacky physicist measures $t_{1/3} = 100$ hours for Whackyonium. Suppose they have isolated 9 $\mu$g of Whackyonium. 200 hours later how much Whackyonium will they have left?

(b) The whacky nuclear physicist has to work with nuclear physicists who prefer the usual half-life measure, $t_{1/2}$. Those physicists use the formula $N(t) = N(0)\left(\frac{1}{2}\right)^{t/t_{1/2}}$. Of course the formulas must agree. Set the two expressions for $N(t)$ to each other and cancel off the $N(0)$ that appears on both sides.

(c) Take ln, the natural log, of the equation you found in (b). Simplify and cancel as much as you can. What is the relationship between $t_{1/2}$ and $t_{1/3}$?

HINT: When you are done, all the minus signs will be gone, and all the fractions will be gone, and you will just have a very simple relation like the one between $t_{1/2}$ and $\tau$.

## 4. The Binding Energy of the Alpha Particle (4 pts)

**NB: Do the remaining nuclear physics problems so that your final answers for energies are in MeV or atomic mass units, u, not SI units, which are darned inconvenient for nuclear physics.**

The alpha particle has two protons and two neutrons, for four nucleons in total.

(a) The two protons repel each other. Assuming that in the alpha particle, the two protons are 1 fm apart and given that $\frac{e^2}{4\pi\epsilon_0} = 1.5\ \text{eV}\cdot\text{nm}$, what is Coulomb energy required to bring these protons to this distance apart?

(b) Each nucleon in the alpha particle is attracted to the other three by the “strong interactions”. How many pairs of nucleons are there?

HINT: Be super-careful not to double-count. (Tosca being on BH with Oleanna is exactly the same as Oleanna being on BH with Tosca. That’s not two different BH assignments.)

(c) Assuming each pair has a binding energy of 5MeV, what is the strong interaction binding energy?

(d) Using your answers to (a), (b), and (c) what is the net binding energy?

(e) From your answer to (d), what is the mass deficit of the alpha particle? To be more precise about what I am asking, how much lighter is a Helium atom than 2 protons, 2 neutrons, and 2 electrons? All you need is your answer to (d) and 1 u = 938 MeV / $c^2$. Answer to four decimal places.

HINT/CROSSCHECK: I fudged the numbers until I got close to the actual value which begins with 0.03.

## 5. Understanding Nuclear Energy Levels (4pts + 1EC)

Moore Introduced lovely diagrams in Chapter 14 to help us understand $\beta^-$, $\beta^+$decays, and electron captures. These diagrams don't help much with understanding alpha decays.

At a deep level, $\beta^-$ decays are,

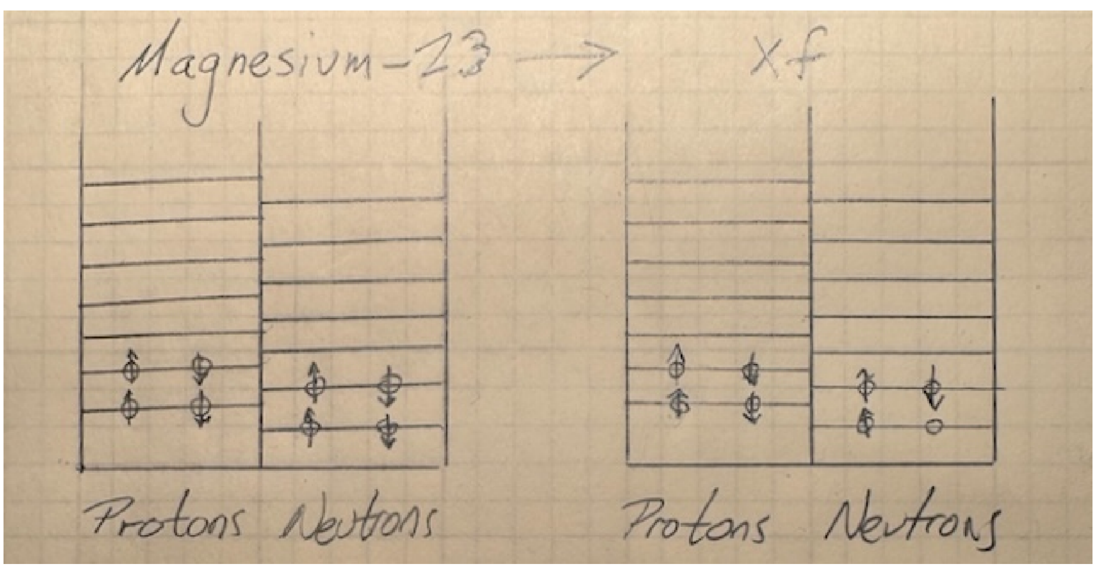
$n \rightarrow p + e^- +$ antineutrino

and $\beta^+$ decays are,

$p \rightarrow n + e^+ +$ neutrino

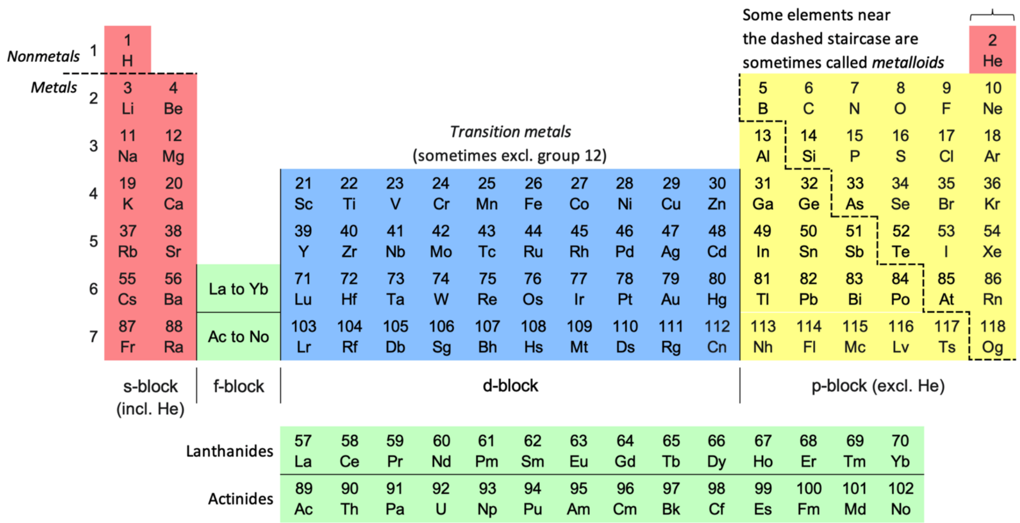
In this problem, we are going to understand what happens to Magnesium-23 using Moore's diagrams. Magnesium is element 12 in the periodic table and has abbreviation Mg. Its complete symbol is therefore $^{23}_{12}$Mg.

(a) In the diagram *on the left* below, I have gotten you started by drawing in 4 of the protons and 4 of the neutrons. Complete the diagram on the left by drawing in the rest of the protons and neutrons in Magnesium-23.




(b) In the diagram *on the right* above, show what you expect Magnesium-23 to decay into by drawing in the rest of the protons and neutrons for Xf.

(c) Consult the periodic table below to determine the actual name of Xf and then write the final result as ${}_{Z}^{A}\mathrm{Xf}$ with the values of $Z$ and $A$ you have now determined.




(d) So, now you can write the entire reaction. Your answer for the complete reaction should look something like ${}_{43}^{99}\mathrm{Tc} \to {}_{44}^{99}\mathrm{Ru} + e^{-}$ + antineutrino, but of course the actual inputs and outputs will be completely different.

(e) EXTRA CREDIT: The reaction you found in (d) is energetically allowed and so that is what happens. In fact, the half-life is only 12 seconds. If the reaction in (d) were not energetically allowed, how would you write the reaction that you would then expect?

## 6. Final Fusion Processes in Large Stars (3 pts)

$$^{28}_{14}\text{Si} + {}^{4}_{2}\text{He} \rightarrow {}^{32}_{16}\text{S}$$
$$^{32}_{16}\text{S} + {}^{4}_{2}\text{He} \rightarrow {}^{36}_{18}\text{Ar}$$
$$^{36}_{18}\text{Ar} + {}^{4}_{2}\text{He} \rightarrow {}^{40}_{20}\text{Ca}$$
$$^{40}_{20}\text{Ca} + {}^{4}_{2}\text{He} \rightarrow {}^{44}_{22}\text{Ti}$$
$$^{44}_{22}\text{Ti} + {}^{4}_{2}\text{He} \rightarrow {}^{48}_{24}\text{Cr}$$
$$^{48}_{24}\text{Cr} + {}^{4}_{2}\text{He} \rightarrow {}^{52}_{26}\text{Fe}$$
$$^{52}_{26}\text{Fe} + {}^{4}_{2}\text{He} \rightarrow {}^{56}_{28}\text{Ni}$$

All of the above reactions occur in the core of a large star near the end of its life. Let's estimate the temperature required for one of these reactions. We'll do the first one, $^{28}_{14}\text{Si} + {}^{4}_{2}\text{He} \rightarrow {}^{32}_{16}\text{S}$.

(a) Estimate the radius, $r_{\text{Si}}$ of the $^{28}_{14}\text{Si}$ nucleus using the empirical formula $r = r_0\, A^{1/3}$ where $r_0 = 1.2$ fm. Also estimate the radius, $r_{\text{He}}$ of $^{4}_{2}\text{He}$ using the same formula.

(b) If the nuclei must just touch to fuse, what is the kinetic energy required to overcome the Coulomb energy? You will need $\frac{e^2}{4\pi\epsilon_0} = 1.5\ \text{eV}\cdot\text{nm}$.

HINT: The answer is not just $\frac{e^2}{4\pi\epsilon_0(r_{\text{Si}}+r_{\text{He}})}$. A lot of people did that on Problem Set 18. If that is all you put into your calculator, you are forgetting something.

(c) Using KE = $\frac{3}{2}\, k_B\, T$ and assuming all the kinetic energy required to overcome the Coulomb energy must be in the lighter nucleus (rather than distributed half and half in the two nuclei), what must the temperature in the interior of the star be? You will need $k_B = 8.6 \cdot 10^{-5}\ \frac{\text{eV}}{{}^\circ\text{K}}$.

COMMENT 1: Once all the fusion processes above occur in the core of a large star, its core is out of energy and the core collapses. Core collapse is sudden, and the supernovae process we discussed back on March 19 (https://brianhill.github.io/physics-ii/resources/FermionGas.nb.pdf) sets in.

COMMENT 2: These temperature estimates we have been doing are always overestimates. If you care, ask me why, and I will explain. It is too much for a comment.

# Name ____________________

1. / 5
2. / 5
3. / 4 + 1EC
4. / 4
5. / 4 + 1EC
6. / 3

GRAND TOTAL

/ 25 + 2EC = 25 MAX