# An Aging Paradox

One expects reciprocity: if I say your meter sticks are foreshortened, then you should say my meter sticks are foreshortened. But how can that be? If I hold two meter sticks side-by-side as they briefly pass, can each be shorter than the other?

Similarly if I say your clocks rub slow, then you should say my clocks run slow. But can't we just directly compare them?

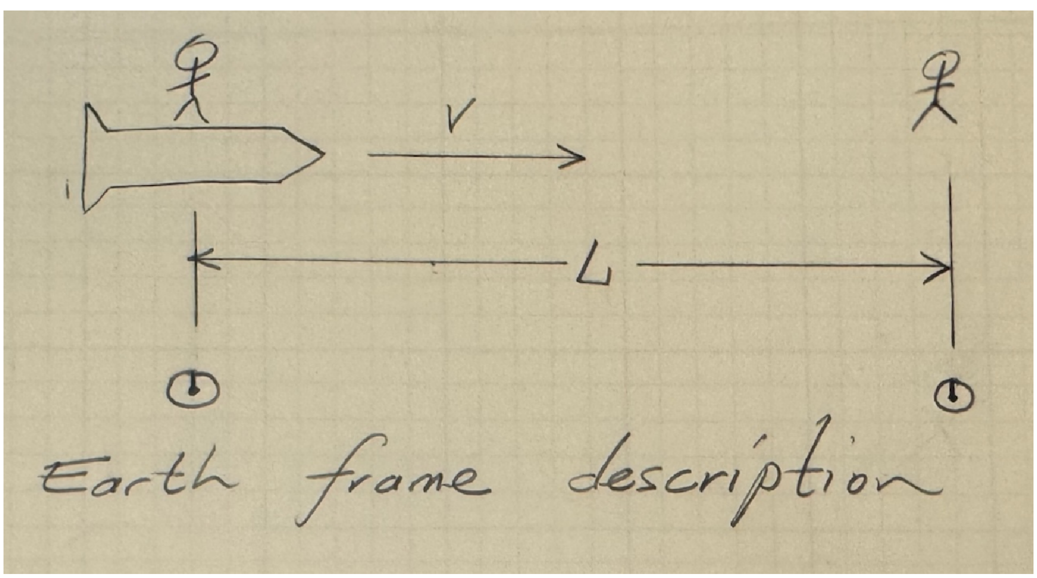
The Relativity of Simultaneity is the way out of these paradoxes and various others.

Let's consider an aging paradox. If we start out the same age, and you are moving relative to me, I say you age more slowly. But can you also say I age more slowly. When we pass each other, one of us has to be older.

Let's carefully consider the two descriptions.

## Earth-Frame Description

Here is the situation, initially in the Earth frame.




Things to notice:

The clocks are synchronized in the Earth frame. The rocket is $L$ away in the Earth frame. The rocket is moving with speed $v$ towards the Earth frame observer.

According to the Earth frame clocks, shown as they appear in the Earth frame, the two people are initially the same age.

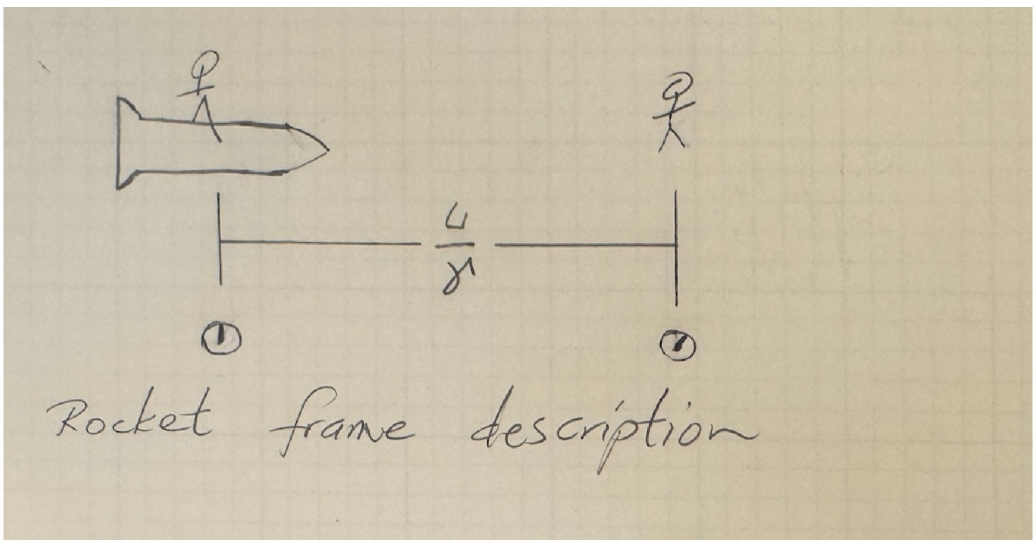
## Earth-Frame Analysis

When we meet (when the rocket-frame observer passes me), I will have aged $\frac{L}{v}$. Your (the rocket frame observer's) clocks run slow, and you age correspondingly slowly, so you will have only aged $\frac{1}{\gamma}\frac{L}{v}$.

Therefore, I will be $\frac{L}{v} - \frac{1}{\gamma}\frac{L}{v}$ older when we meet.

## Rocket-Frame Description

Here is the situation, initially in the Rocket frame.




The #1 thing to notice:

The rocket ship observer says, I see you are making a measurement with a meter stick that you assert is $L$ long. That meter stick is foreshortened. You are only $\frac{L}{\gamma}$ away from me initially.

## Rocket-Frame Analysis

When we meet (when I pass you), I will have aged $\frac{L}{\gamma v}$. You (the Earth frame observer) are the one whose clocks run slow! While I have aged $\frac{L}{\gamma v}$, you will have aged $\frac{1}{\gamma}\frac{L}{\gamma v}$. So I will have aged $\frac{L}{\gamma v} - \frac{1}{\gamma}\frac{L}{\gamma v}$ more!

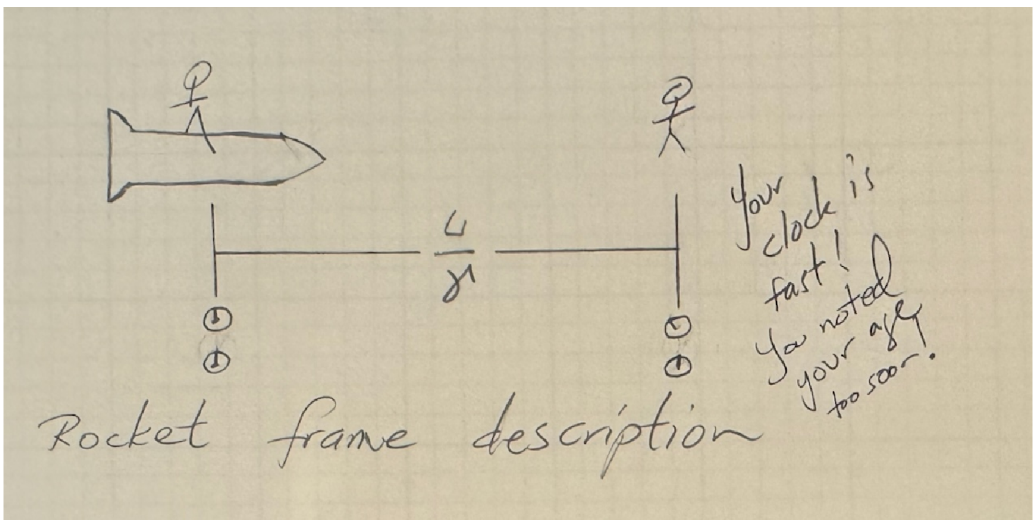
## The Paradox

It must be, when they briefly meet, that as they pass, they can briefly compare ages. They each cannot be older than the other.

## The Relativity of Simultaneity

Einstein's relativity of simultaneity says that observers can even disagree about what things are simultaneous. In particularly, they can disagree about whether clocks are synchronized. The Earth-frame observer said that initially (when the rocket ship was $L$ away), the clocks were synchronized, and the two people had the same age according to the two clocks.

The rocket ship observer says the initial situation was like this:




Notice I have drawn two sets of clocks. The lower clocks are the Earth observer's clocks. The upper clock are the rocket ship observer's clocks. Notice the disagreement I have put in.

The rocket frame observer says when you were $\frac{L}{\gamma}$ away from me, the clock you had with you was $\frac{v}{c^2} L$ ahead of what it should be. And you claimed we were the same age when you looked at that clock, but because your clock was fast, you noted your age too soon.

You need to add the amount of clock disagreement, which is $\frac{v}{c^2} L$, to your initial age.

# Summary/Reconciliation

## Earth Frame

We were the same age when we were $L$ apart, and I am $\frac{L}{v}\left(1-\frac{1}{\gamma}\right)$ older when we meet.

## Rocket Frame

You were already $\frac{v}{c^2} L$ older when we were $\frac{L}{\gamma}$ apart. After that, I aged $\frac{L}{\gamma v} - \frac{1}{\gamma}\frac{L}{\gamma v}$ more. So when we meet, you are still

$$\frac{v}{c^2} L\text{-}\left(\frac{L}{\gamma v} - \frac{1}{\gamma}\frac{L}{\gamma v}\right)$$

older. Let's factor out $\frac{L}{v}$:

$$\frac{L}{v}\left(\frac{v^2}{c^2} - \left(\frac{1}{\gamma} - \frac{1}{\gamma^2}\right)\right) = \frac{L}{v}\left(\frac{v^2}{c^2} - \left(\frac{1}{\gamma} - \left(1 - \frac{v^2}{c^2}\right)\right)\right) = \frac{L}{v}\left(\frac{v^2}{c^2} - \left(\frac{1}{\gamma} - \left(1 - \frac{v^2}{c^2}\right)\right)\right) = \frac{L}{v}\left(1 - \frac{1}{\gamma}\right)$$

Perfect agreement! Both observers say that the Earth-frame observer is $\frac{L}{v}\left(1-\frac{1}{\gamma}\right)$ older when they meet.