# Critical Properties of the Electron

*Not only does the electron have position and momentum, but it also has angular momentum and a magnetic moment.*

## Position

Classically at least, the electron has a position. You will see that the classical idea of position is muddied by quantum mechanics, but give that time for Moore to develop. For the present it is important just to know that classically, the electron has a position.

The position is usually given by the three coordinates, $x$, $y$, and $z$, and these are often combined into a vector, $\vec{r}$.

## Momentum

The electron also has a momentum. The classical idea of momentum is also muddied by quantum mechanics, but also give that time to develop. In non-relativistic mechanics, the momentum is just the mass times the velocity. So you might ask, why not say that the electron has a velocity? Rest assured, it is more convenient to work with the momentum than the velocity.

The momentum is usually given by its three components, $p_x$, $p_y$, and $p_z$. These are often combined into a vector, $\vec{p}$. The kinetic energy of the electron in terms of its velocity is $\frac{1}{2}mv^2$, where $v \equiv |\vec{v}|$, and $v^2 = v_x^2 + v_y^2 + v_z^2$, but since it is more convenient to work in terms of its momentum, you will usually see a particle's kinetic energy written as $\frac{p^2}{2m}$ where $p \equiv |\vec{p}|$ and $p^2 = p_x^2 + p_y^2 + p_z^2$.

## Spin

You can think of the electron as a little sphere. Thinking this way turns out to be useless, except for one thing: if it is a little sphere, then it could be spinning!

Let's, for a minute, indulge this uselessness, and think of it as a sphere with its mass uniformly distributed throughout the sphere. If it is spinning with angular speed $\omega$, then back in Volume C, Section C6.4, p. 89, we learned that $L = I\omega$, where $I = \frac{2}{5}mr^2$, $m$ is the mass of the sphere, $r$ is the radius of the sphere, and $L$ is the magnitude of the angular momentum vector, $L \equiv |\vec{L}|$.direction that $\vec{L}$ points is determined by the right-hand rule. Sometimes we write $\vec{L} = I\vec{\varpi}$ and say that the angular velocity is itself a vector, $\vec{\varpi}$. Moore had a drawing of the application of the right-hand rule on p. 84 of Volume C:

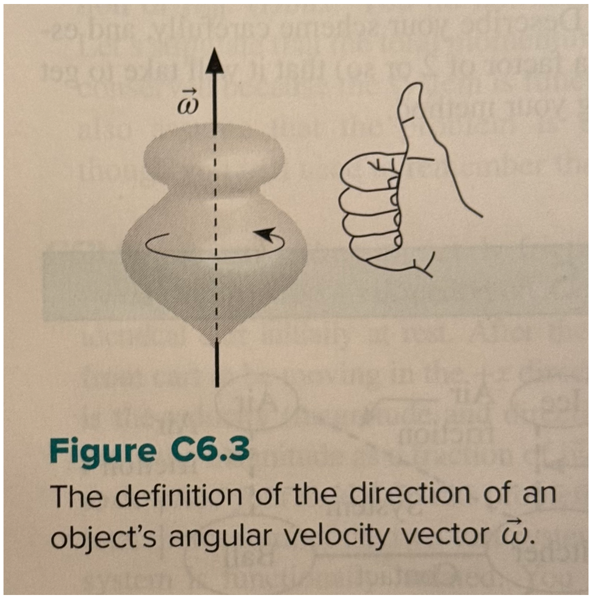

**Figure C6.3**
The definition of the direction of an object's angular velocity vector $\vec{\omega}$.

Since $\vec{L} = I\vec{\varpi}$ is a vector, it has three components, $L_x$, $L_y$, and $L_z$. Because this little bit of angular momentum always goes along with the electron itself and has nothing to do with the angular momentum it might have by being in a circular orbit around something else (like the Sun, or a proton), we give this little bit of angular momentum its own name, spin, and its own vector and symbol $\vec{S}$, and $\vec{S}$ has three components $S_x$, $S_y$, and $S_z$.

## Summary So Far

So far, you know that the electron has position, momentum, and spin. But it also has a charge $-e$. So in the next section, we deal with that.

## Torque

You really need to review torque. I am trying to hit high points in this document so that you don't lose the forest for the trees. I am not trying to recreate the whole textbook. The most important fact about torque $\vec{\tau}$ was that

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

This makes it quite analogous to the equation

$$\vec{F} = \frac{d\vec{p}}{dt}$$

which just the formula for non-relativistic momentum and the definition of acceleration is a sweet way of re-writing the first (or second) most famous equation in all of physics:

$$\vec{F} = m\vec{a}$$

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# Magnetic Moment

If a little sphere is charged and it is spinning, then it is also a little magnet! We quantify the strength and direction of the little magnet, in a quantity we call the magnetic moment, which in symbols is $\vec{\mu}$ .

Because the electron is negatively charged, the direction its north pole points is exactly opposite the direction that its angular momentum vector points. We capture this in equations as follows:

$$\vec{\mu} = g \frac{-e}{2m} \vec{S}$$

The factor $g$ out front captures our ignorance of the proportionality constant.

If the charge on the electron is closer to the surface of the sphere, than the mass, then $g$ will be greater than 1. If it is closer to the center of the sphere than the mass, then $g$ will be less than 1.

This $g$ has absolutely nothing to do with the acceleration of gravity at the surface of the Earth. It is just a number. It is the single most accurately-measured number in all of science. Its value is:

$g = +2.00231930436256\,(35)$

where the (35) at the end refers to the 1-sigma uncertainty in the last two digits, and the minus sign that you sometimes see in $g$ has been put in the equation for $\vec{\mu}$.

I can’t leave the subject of the magnetic moment of the electron without saying that this is also the most accurately-predicted number in all of science.

At present, theory and experiment agree to within each other’s mutual uncertainty, and in fact, it is theory that has about six times the uncertainty than the experiment, due to the extreme difficulty of the theoretical calculation.

## Summary

We have appealed to the idea that the electron is like a little spinning sphere. If it were, then it isn’t a surprise that it has some angular momentum and some magnetic moment.

However, the idea is useless. No radius $r$ of the electron has ever been detected. It is for all practical purposes a point, and the theory that predicts $g$ to such tremendous accuracy treats it as a point.

So it is better to think of the electron as a Euclidean idealization of an object, and this Euclidean idealization also has some angular momentum and is a little tiny magnet.

There is so much interesting behavior in these two facts that in Chapter C6, we will almost ignore the fact that the electron also has position and momentum.

You will be studying beams of electrons, and the the single most interesting thing about these beams is their angular momentum and their magnetism.[1]

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[1] In the next section you will read, Q6.4, you will see that this sentence is an oversimplification. It is hard to make beams of electrons. It is possible, but the original experiments used beams of silver atoms. Grant the oversimplification. It is almost as if we are dealing with a beam of electrons. That's because 46 of the 47 electrons in a silver atom are so tidily paired up into groups of 2 with opposite spin and magnetic moment that the grand effect of those 46 is that they have no net spin or magnetic moment. Only the spin and angular momentum of the electron that is not paired up matters.