Physics, Preparation for Friday, Oct. 27

Read N1 from *Six Ideas*

For Problem Set #7

1. Continuation of C14M.10, p. 237

There is enough going on in this problem that it is easy to lose the forest for the trees, so let's be very organized...

To make the variable names memorable, let's do the following: For the Frisbee, which is both spinning and moving, we'll use lower-case variables:

$m,\ \vec{v},\ i,\ \vec{\omega}$

We also have $i = \alpha m r^2$ for the Frisbee.

For the astronaut, who initially is neither spinning nor moving, we'll use upper-case variables:

$M,\ I$

We also have $I = AMR^2$ for the astronaut. We could put in that $\alpha = \frac{2}{3}$ for the Frisbee, and $A = \frac{1}{2}$ for a cylindrical astronaut, but we can also leave α and A as symbols so that we don't lose where the 2's and 3's came from.

For the combined Frisbee-astronaut system after the catch, which is spinning and moving at slower rates because the angular momentum and linear momentum have been distributed between the astronaut and the Frisbee, the relevant variables are:

$m + M,\ \vec{V},\ I + i,\ \vec{\Omega}$

Finally, we have Moore's suggestion to introduce dimensionless ratios:

$b \equiv \frac{m}{M}, \quad \rho \equiv \frac{r}{R},$ and $|\vec{\omega}| = u\,\frac{|\vec{v}|}{r}$.

The last of those three equations serves to define u. Roughly speaking if u is big, the Frisbee is "spinning more than it is moving," and if u is small, the Frisbee is "moving more than it is spinning." Think about it until the rough description actually makes sense

(a) We found (using conservation of ordinary momentum) that:

$$\vec{V} = \frac{m}{m+M}\,\vec{v} = \frac{b}{b+1}\,\vec{v}$$

(b) We found (using conservation of angular momentum) that:

$$\vec{\Omega} = \frac{i}{i+I}\,\vec{\varpi} = \frac{\alpha m r^2}{\alpha m r^2 + A M R^2}\,\vec{\varpi} = \frac{\alpha b \rho^2}{\alpha b \rho^2 + A}\,\vec{\varpi}$$

(c) We calculated K_i, where K_i is the sum of the initial translational and rotational kinetic energy in the Frisbee. It was:

$$K_i = \frac{1}{2}\,m\,|\vec{v}\,|^2 + \frac{1}{2}\,i\,|\vec{\varpi}|^2 = \frac{1}{2}\,m\,|\vec{v}\,|^2 + \frac{1}{2}\,\alpha m r^2\,|\vec{\varpi}|^2 = \frac{1}{2}\,m\,|\vec{v}\,|^2 + \frac{1}{2}\,\alpha m r^2\left(u\,\frac{|\vec{v}\,|}{r}\right)^2 = \frac{1}{2}\,m\,|\vec{v}\,|^2\left(1+u^2\right)$$

We calculated K_f. It was:

$$K_f = \frac{1}{2}\,(m+M)\,|\vec{V}\,|^2 + \frac{1}{2}\,(i+I)\,|\vec{\Omega}\,|^2 = \frac{1}{2}\,(m+M)\left(\frac{b}{b+1}\right)^2\,|\vec{v}\,|^2 + \frac{1}{2}\left(\alpha m r^2 + A M R^2\right)\left(\frac{\alpha b \rho^2}{\alpha b \rho^2 + A}\right)^2\left(u\,\frac{|\vec{v}\,|}{r}\right)^2$$

That was as far as we got on the blackboard.

I put some pale magenta and green highlighting in the equation above. As a step to making K_f a tad nicer, how about taking the green-highlighted r in the denominator and replacing it with ρR? Then use the R that is now in the denominator (squared, actually), to simplify $\alpha m r^2 + A M R^2$.

Moore asked us to compute the fractional loss for part (c). So your final step for part (c) is to make

$$\frac{K_i - K_f}{K_i}$$

look as nice as you can. By systematically getting rid of m and r in favor of M, R, b, and ρ you should be able to get the fractional loss written entirely in terms of the dimensionless numbers, b, ρ, u, α, and A.

(d) Finally, we plug in Moore’s values, but the way to do it is to compute the dimensionless ratios b, ρ, and u, (and now use $\alpha = \frac{2}{3}$ and $A = \frac{1}{2}$ and then use your nice expressions for $\vec{V}$, $\vec{\Omega}$, and $\frac{K_i - K_f}{K_i}$ that only involve dimensionless ratios.

2. A Slingshot Problem

As our slingshot problem, how about you just study C14.4 and do Exercise C14X.2?

Then let's apply this formula to a slingshot of a spacecraft off of Venus. The orbital speed of Venus is 126,000km/hr. Assume your rocket can achieve a speed of 58,000km/hr using its own thrust (which is the fastest a chemical rocket has ever gone).

What speed can it achieve after being slingshot off of Venus?

3. N1B.5

4. N1B.11

5. N1M.3