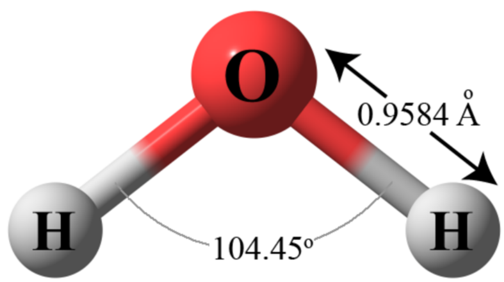
Modern Introductory Physics Midterm

Tuesday, Oct. 10, 2023 — Covering Most of *Six Ideas,* Volume C, Chapters C1-C11

1. Center of Mass (3 pts)

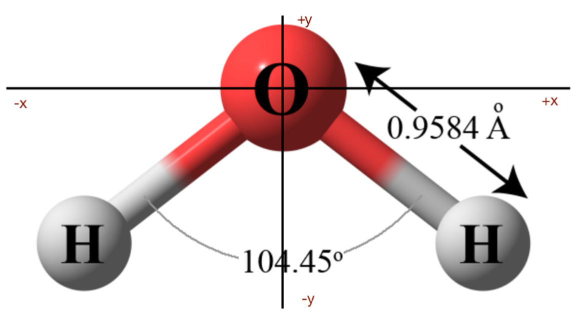


A molecule of water is shown. Let's give the numbers in the drawing some symbolic names:

$\alpha = 104.45\,^{\circ}$ and $D = 0.9584$ Å

The unit Å is the "Angstrom," and it is 10^{-10} m.

Let's set up a coordinate system. Put the Oxygen at $x = y = 0$, and put the two Hydrogens in the minus-y direction, like this



Without plugging in the numerical values (e.g., just using the symbols D and α), and also m_{H} (the mass of each of the two Hydrogens), and m_{O}(the mass of the Oxygen), compute $\vec{r}_{\mathrm{CM}}$.

You can put your answer in any form that makes it clear that you are computing a vector, which means you need to compute both its length and its direction, or alternatively, its components.

Your derivation should start with the definition of $\vec{r}_{\mathrm{CM}}$, which is:

$\vec{r}_{\mathrm{CM}} \equiv \frac{1}{M}\left(m_1\,\vec{r}_1 + m_2\,\vec{r}_2 + \ldots + m_N\,\vec{r}_N\right)$ (*Six Ideas,* Eq. C4.1)

2. Conservation of Momentum (4 pts)

The Law of Conservation of Momentum is:

$\vec{p}_{1f} + \vec{p}_{2f} + \ldots \equiv \vec{p}_{1i} + \vec{p}_{2i} + \ldots$ (*Six Ideas,* Eq. C5.1)'

In words, this equation says that the total momentum after an interaction has occurred is equal to the total momentum before it occurred.

Use conservation of momentum to solve this problem:

C5M.6 During the filming of a certain movie scene, the director wants a small car (whose mass is m_1) traveling due east at speed $|\vec{v}_1|$ to collide with a small truck (whose mass is m_2) traveling north. The director also wants the collision to be arranged so that just afterward, the interlocked vehicles travel straight toward the camera. If the line between the camera and collision makes an angle of θ with respect to north, at what speed $|\vec{v}_2|$ should the trucker drive?

To clarify, Moore is describing a problem where m_1, m_2, $|\vec{v}_1|$ and θ are given and he wants you to find $|\vec{v}_2|$ in terms of them.

Begin your solution by making a diagram of what is being described and include in the diagram the coordinate system you choose to solve the problem.

3. Twirl and Torque (4 pts)

Below is an equation you haven't used much, but it is an important one, and it was part of the material we covered. The equation is the definition of the torque vector:

$\vec{\tau} \equiv \frac{\Delta \vec{L}}{\Delta t}$ (*Six Ideas,* Eq. C6.13, simplified)

In the numerator on the right-hand side is the change in angular momentum. In the denominator is the change in time. ***All you are going to do in this problem is compute $\Delta \vec{L}$.***

Here is the initial situation:

Initially you are bicycling on level ground a small angle β (changed from α to avoid confusion) east of north at a speed $|\vec{v}|$. *Concentrating on the front wheel wheel only (don't worry about your own momentum or the rest of the bike),* assume that the front wheel has mass M and radius R, and that all of its mass is in its edge (and none of the mass is in the wheel's spokes or hub), which means that you can treat the wheel as a hoop:

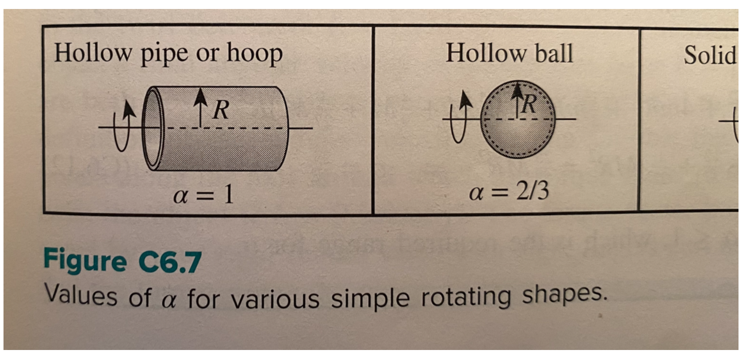


Figure C6.7
Values of α for various simple rotating shapes.

(a) The first step in is to make a diagram. Make your diagram a top view of the rotating wheel with z coming out of the page. *Make north the +x direction and west the +y direction.* Label β, the angle from north to show the direction the wheel is rolling. Use the right-hand rule to reason about and label the initial angular momentum, $\vec{L}_i$.

(b) Later (finally), you are bicycling β to the west of north at the same speed $|\vec{v}|$, just in a new direction. Draw another top-view diagram that shows the new situation for the rotating wheel. Label $\vec{L}_f$, the final direction of the angular momentum.

(c) In (a) you drew $\vec{L}_i$. In (b), you drew $\vec{L}_f$. Now make a third diagram that shows $\Delta \vec{L} = \vec{L}_f - \vec{L}_i$.

(d) You have two sides of a triangle specified in (c) and (d). Use trig to discover the magnitude of the third side, $\Delta \vec{L}$ in terms of the givens M, R, $|\vec{v}|$, and β.

4. Conservation of Energy (4 pts)

Do Problem C8M.6:

C8M.6 You are trying to design a "rail gun" to launch canisters with mass m at a high speed $|\vec{v}_i|$ on an initially almost horizontal trajectory along the surface of the moon (whose mass is M and radius is R). If $|\vec{v}_i|$ is large enough, the canisters will fly tangentially away from the moon into deep space. You want the canisters to have a certain speed $|\vec{v}_f|$ once they are very far from the moon.

(a) What should $|\vec{v}_i|$ be in terms of M, G, R, and $|\vec{v}_f|$?

(b) Calculate the numerical value of $|\vec{v}_i|$ for $|\vec{v}_f| = 500$ m/s, given that the moon's radius is 1740 km and its mass is 7.36×10^{22} kg.

You also need $G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$.

XXX. Work

Work is a very important concept, but we have done lots of work problems (crane swinging, fail-army-style car collisions, etc.), and this exam is already getting long, so I'm going to skip over having a work problem.

5. Rotational Energy (5 pts + 1pt EC)

Do Problem C11M.6:

C11M.6 A bicyclist coasts from rest down a hill. The rider and cycle together have a mass M, and each wheel has a mass $m = M/50$ and an α of $2/3$.

(a) Assuming no friction, what is the cyclist's speed after going down a vertical distance h?

(b) What fraction of the bike and rider's total energy of motion is rotational energy in the wheels?

HINT1: You should identify five sources of energy: (1) the translational energy of the front wheel, which has mass m, (2) the rotational energy of the front wheel, (3) the translational energy of the back wheel which also has mass m, (4) the rotational energy of the back wheel, and (5), the translational energy of the rest of the rider-bicycle system, which has mass $M - 2\,m$.

HINT2: It is going to make a mess to have factors of 1/50, 48/50, and 2/3 flying around the problem. To avoid a mess, just leave everything in terms of M, m, α, and $g \cdot h$.

HINT3: There is only one speed in this problem. In (a), you need to use conservation of energy to find it. Your answer for the speed will involve M, m, α, and $g \cdot h$. If your answer has something else in it like ω, figure out how to get rid of it.

EXTRA CREDIT: If you have extra time, put in $m = M/50$, $M - 2\,m = 48\,M/50$, and $\alpha = 2/3$ and compute the ratio Moore asked for.

Name ____________________

1. / 3
2. / 4
3. / 4
4. / 4
5. / 5 (+1 EC)

GRAND TOTAL

/ 20 MAX