Modern Introductory Physics Final

Tuesday, Dec. 12, 2023 — Covering Most of *Six Ideas,* Volume N, Chapters N1-N10

1. Statics (5 pts)

This pole vaulter is running down the track at a fast but constant speed and direction. Let's assume the pole is perfectly level even though it is tilted up slightly in the photo. Because the pole's velocity and orientation is constant, you can analyze the forces on the pole with statics.

(a) Model the pole vaulter's right hand as being at a distance a from the back end of the pole. *The pole vaulter is facing us, so what we call his right hand is what we see in the photo as the hand on the left.* Model the pole vaulter's left hand as being a distance b from the back end of the pole. The pole's total length is c and the pole has a uniformly distributed mass m. Draw a nice free body diagram for the pole that shows all three forces on the pole: $\vec{F}_R$ (the right hand), $\vec{F}_L$ (the left hand), and $\vec{F}_G$ (gravity).

1. Statics (CONT'D)

(b) What does conservation of momentum say about these three forces? Just write down the vector equation: ... + ... + ... = ... applicable for statics.

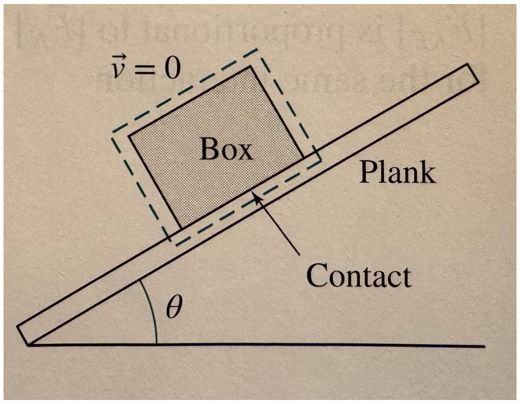
(c) Assume that the forces the pole vaulter's hands are applying to the pole are upwards or downwards only. Because all three forces are only up and down, there is only one significant component of the vector equation your wrote down in (b). Let's call the vertically upwards direction the $+y$ direction. Write down the conservation of momentum equation in terms of $F_{R,y}$, $F_{L,y}$, and $F_{G,y}$. Feel free to already use the fact that $F_{G,y} = -mg$. Then solve this equation for $F_{L,y}$.

(d) Conservation of angular momentum says that the three torques must add up to zero. You can analyze the torques from any point along the pole, but let's not be tricky. Let's just use the back end of the pole for the analysis. Write down an equation involving $F_{R,y}$, $F_{L,y}$, and $F_{G,y} = -mg$, and a, b, and c. The equation you write down is a consequence of $\tau_{\text{total},z} = 0$ where the z-direction is out of the page.

(e) Use the equation for $F_{L,y}$ you got in (c) to get rid of $F_{L,y}$ in the equation you got in (d). Solve for $F_{R,y}$.

(f) Plug in $m = 3$ kg, $g = 10\,\text{m}/\text{s}^2$, $a = 0.5$ m, $b = 1.0$ m, and $c = 4.0$ m. If you did the algebra right, you'll get an answer for $F_{R,y}$ that is negative and larger in magnitude than mg!

2. Friction and Inclined Planes (4 pts)



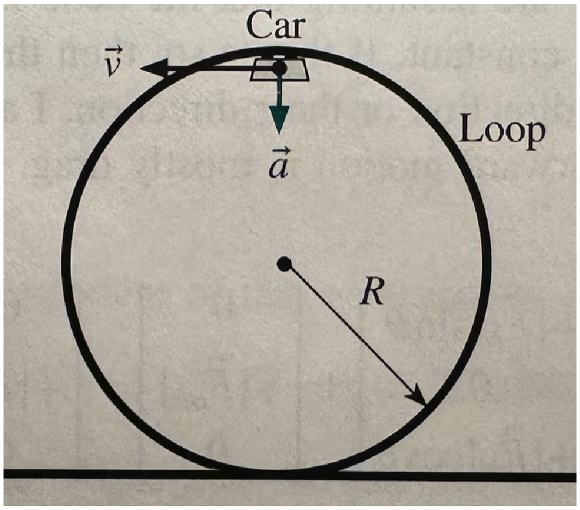
This is another statics problem, but it has static friction and some angles. Imagine that the angle θ is slowly and gently increased (this is still a statics problem because the rate of increase is slow and gentle). At some angle θ_{MAX} the box is going to start sliding. If μ_S is small (such as for a steel box on a greasy plank), that will happen for a shallow angle. If μ_S is big (such as for a rubber-coated box sitting on a granite plank), the box will start sliding at a steep angle. You are going to find μ_S.

(a) Draw a nice free body diagram for the box that shows the three forces acting on the box. Note somewhere what you are using as the $+x$ direction and the $+y$ direction. This is a 2-d problem, so there is no need to consider the z direction.

(b) Write down the vector equation that governs the situation when $\theta = \theta_{MAX}$ and $|\vec{F}_S| = |\vec{F}_{S,MAX}| = \mu_S |\vec{F}_N|$. Do the equation as an equation with column vectors with the components of your column vectors written in terms of vector magnitudes and trig functions.

(c) What nice simple expression does the x-component of this equation give for μ_S?

3. Circular Motion (5 pts)



A roller coaster car is going around a loop.

(a) Draw a free body diagram for the car at the top of the loop. You don't have to include friction or drag, so there are only two forces to put on your diagram. Just so we all have similar diagrams, let's have $|\vec{v}|^2/R = 1.25\,g$. To help you make your diagram realistic, I'll tell you that the consequence of that choice is that $\vec{F}_N$ should only be one-quarter as long as the vector $\vec{F}_G$. In fact, so that we have even more similar diagrams, I will pass out rulers. In your free body diagram, make $\vec{F}_G$ be 1 inch long and $\vec{F}_N$ be 1/4 inch long.

(b) Write down equations involving $|\vec{F}_{\text{N}}|$, mg, and $|\vec{v}|^2/R$. Solve for $|\vec{F}_{\text{N}}|$. Use that we assumed $|\vec{a}| = |\vec{v}|^2/R = 1.25\,g$.

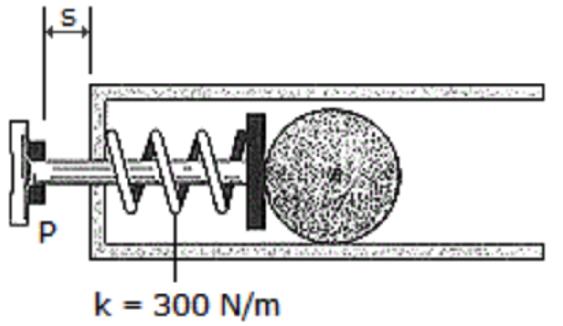
(c) What would have happened in the equation for $|\vec{F}_{\mathrm{N}}|$ if $|\vec{v}|^2/R < g$? What does this mean? HINT: The theme park management, their lawyers, and the stockholders do not want this to happen.

(d) Draw the car again on the original picture on p. 4, but at the bottom of the loop. Obviously it is going faster at the bottom, so make that obvious by making the new $\vec{v}$ twice as long as the original $\vec{v}$. The original $|\vec{a}|$ was 1.25 g. What is the new $|\vec{a}|$? Use that magnitude to put in the new $|\vec{a}|$ accurately.

(e) Draw another free body diagram for the car at the bottom of the loop. I specified the sizes of the force vectors in part (a). In this part, make the sizes of the normal force and the gravitational force accurate relative to each other and the sizes I specified in part (a).

(f) Write down the equations involving $|\vec{F}_N|$, mg, and $|\vec{a}| = |\vec{v}|^2/R$ that are applicable at the bottom of the loop. Solve for $|\vec{F}_{\mathrm{N}}|$. Use what you found out about the new $|\vec{a}|$ in part (d). Adjust the $|\vec{F}_{\mathrm{N}}|$ you drew in (e) if necessary.

4. Pinball Launch Spring (6 pts)



The firing mechanism for a pinball in a pinball machine looks something like the above. You pull back the plunger, P, an amount, *s*, and that compresses the spring by that amount. Then you release the plunger and the spring shoves the ball out onto the pinball table. Let's not worry about the fact that the ball rolls. We'll just imagine it has mass *m* and it is shoved out like a projectile.

(a) In the diagram above the value of *k* is given as 300 N/m. If the plunger is pulled back 5 cm, how many Newtons of force does that require?

(b) Let's describe the position of the ball by a variable *x*, and measure *x* from the equilibrium position. So we have $x(0) = -s$. What is $v_x(0)$ if you hold the plunger steady and release it at $t = 0$?

$v_x(0) =$

(c) Two solutions of the harmonic oscillator problem are $A \sin \omega t$ and $B \cos \omega t$. Perhaps it is a surprise, because we haven't had time to mention it, that $x(t) = A \sin \omega t + B \cos \omega t$ is also a solution! Show that this $x(t)$ is a solution of

$$m \frac{d^2 x}{dt^2} = -k x$$

for any values of *A* and *B* provided that $\omega^2 = \frac{k}{m}$. You'll have to take one derivative of it to get $v_x(t)$ and then another derivative to get $a_x(t)$.

(d) For the solution $x(t) = A\sin\omega t + B\cos\omega t$. What is $x(0)$?

(e) When you were taking derivatives in part (c), you first got $v_x(t)$. Plug in $t = 0$ to $v_x(t)$. What is $v_x(0)$?

(f) Compare what you got in (d) and (e) with what you had for $x(0)$ and $v_x(0)$ in step (b). What must A and B be for our the pinball launch situation?

(g) The pinball leaves the plunger when the plunger has pushed it to its equilibrium position. At what time t does the plunger reach the equilibrium position? In other words, $x(t)$ starts out negative. When does $x(t)$ get to zero?

Congrats on getting this far! You deserve a great break!

Name ____________________

1. / 5
2. / 4
3. / 5
4. / 6

TOTAL

/ 20