Distance from Absolute Magnitude

In this unit we introduced a new concept, “absolute magnitude.” The old magnitude that we used is now going to be called “apparent magnitude.”

The essential definition:

Absolute magnitude is the magnitude a star would appear to have if it were brought to a standard distance of 10 parsecs.

We will use M for absolute magnitude and m for apparent magnitude.

Why are we introducing this new quantity?

It is because we have pushed geometry as far as it can be pushed. The angles are getting too small to use the tangent or pie-crust formula to determine distances. The problem is that the angles are just unmeasurably small even once you get as far as the center of our galaxy. We need to get measurements of the distance to the center of our galaxy. We need to get measurements of the distance to other galaxies.

(The best parallax angle measurements — currently the best is the ESA GAIA and Hipparcos satellites — are fractions of a milli-arcsecond. This corresponds to distances of 1000s of parsecs. The quality of the measurement depends on various things, including the magnitude of the star. In round numbers, let’s say that parallax measurements can take us to 1000s of parsecs or many 1000s of light-years.)

Luminosity

We reviewed luminosity, which is another name for power, which we had many weeks back, for example, when we were calculating how much power can be produced by a solar panel. Review:

- *The power output of a star is called its “luminosity.”*
- *The units of power are energy per unit time.*
- *In common scientific units, that is Joules / second. Joules per second is so common in physics it has its own name: “Watts!” abbreviated just W.*
- *In equations, the luminosity of the Sun might be written like:*

$$L_{\text{Sun}} = 3.8 \times 10^{26}\ \frac{\text{J}}{\text{s}} = 3.8 \times 10^{26}\ \text{W}$$

Intensity

- *Intensity is the technical term that corresponds to brightness. Brightness is a little vague, because in common English, it can correspond to either luminosity or intensity. For consistency, I try to stick to these terms: luminosity, intensity, absolute magnitude, and apparent magnitude.*
- *Intensity is energy per unit time* ***per unit area***
- *In common scientific units, that is* Joules / s / m^2 *or W* / m^2
- *In written English, I'd say the preceding as Joules per second per square meter" or "Watts per square meter."*
- *In equations, it might look like:*

  $$I_{\text{Sun at Earth}} = 1367\ \frac{\text{W}}{\text{m}^2}$$

- *The formula for the intensity of the Sun at the Earth is:* $I_{\text{Sun at Earth}} = \frac{L_{\text{Sun}}}{4\pi R^2}$. In this formula, R is the distance from the Earth to the Sun.
- If your telescope has area a, then the power it would collect if you pointed it at a star with luminosity L and distance R is $P = \frac{L}{4\pi R^2}\, a$.

Comparing Intensities of Stars (Brightness Ratio)

$$P_A = \frac{L_A}{4\pi {R_A}^2}\, a$$

$$P_B = \frac{L_B}{4\pi {R_B}^2}\, a$$

We can ask "How bright is Star A compared to Star B?" If you want to get precise, the questioner is asking you to calculate the ratio:

$$\frac{P_A}{P_B} = \frac{\frac{L_A}{4\pi {R_A}^2}\, a}{\frac{L_B}{4\pi {R_B}^2}\, a} = \frac{L_A}{L_B}\, \frac{R_B^2}{R_A^2}$$

From the apparent magnitude, we have another formula for the brightness ratio:

$$\frac{P_A}{P_B} = 100^{(m_B - m_A)/5}$$

We also have an approximate version of the same formula, if you prefer:

$$\frac{P_A}{P_B} = 2.5^{m_B - m_A}$$

The latter formula is close and just comes from using that $100^{1/5}$ is about 2.512.

Since we have all these formulas for brightness ratios, this means we have:

$$2.5^{m_B - m_A} = \frac{L_A}{L_B} \frac{R_B^2}{R_A^2}$$

Comparing Intensities of the same Star at Different Distances

Suppose m_B was the star’s intensity at its true distance, and M_B was the magnitude of the star at 10 parsecs (the “absolute magnitude.”

$$2.5^{m_B - M_B} = \frac{L_B}{L_B} \frac{R_B^2}{(10\text{ parsec})^2} = \frac{R_B^2}{(10\text{ parsec})^2}$$

It’s the same star, so L_B canceled out.

That means you can solve this for the true distance:

$$R_B = \sqrt{2.5^{m_B - M_B}} \times 10\text{ parsecs}$$

Or the more exact version:

$$R_B = \sqrt{100^{(m_B - M_B)/5}} \times 10\text{ parsecs} = 10^{(m_B - M_B)/5} \times 10\text{ parsecs}$$

An Example

$$R_B = 10^{(m_B - M_B)/5} \times 10\text{ parsecs}$$

Apparent magnitude $m_B = 4$ and absolute magnitude $M_B = 1$.

$$R_B = 10^{(4-1)/5} \times 10\text{ parsecs} = 10^{3/5} \times 10\text{ parsecs} = 39.8\text{ parsecs}$$

BOTTOM LINE: If you know a star’s absolute magnitude (somehow, that’s pretty hard), and you know its apparent magnitude (that’s easy, you just look at it through a telescope), then you know its distance!

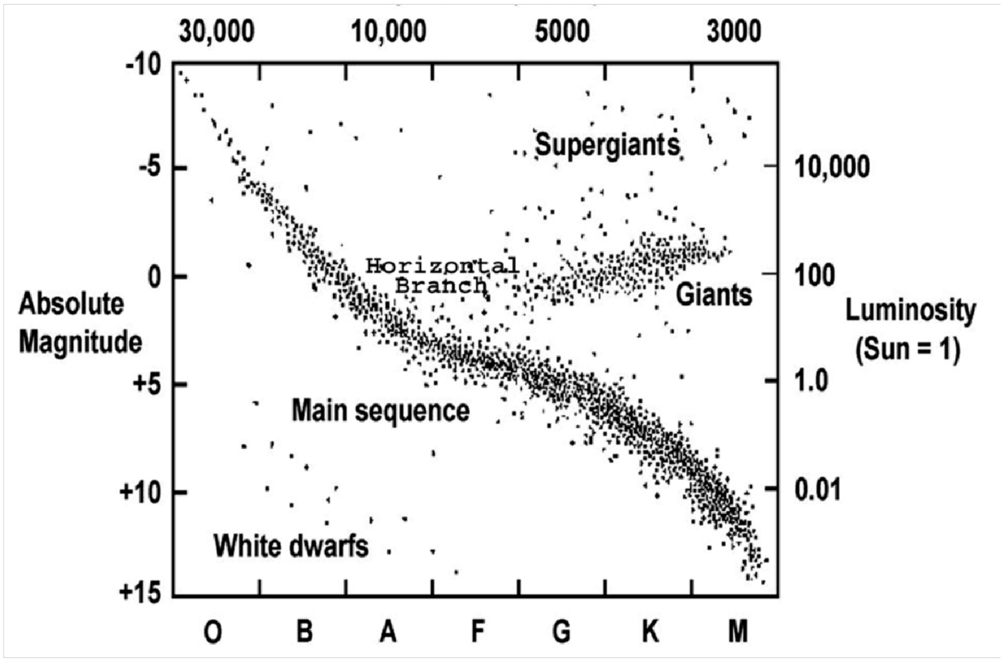
Hertzsprung Russell Diagram

Between 1910 and 1913, Hertzsprung and Russell used the parallax distance and the apparent magnitude of the nearby stars to determine many absolute magnitudes.

They put absolute magnitude (equivalent to luminosity) on one axis.

On the other axis, they put the spectral type or temperature of the star.

The result, shown below is called the Hertzsprung-Russell diagram. Across the top is the temperature in Kelvin, with highest temperature on the left.



Distances from the Hertzsprung-Russell Diagram

On 5/3/19, we derived:

$$2.5^{m_B - m_A} = \frac{L_A}{L_B} \frac{R_B^2}{R_A^2}$$

or if you prefer the more exact version:

$$100^{(m_B - m_A)/5} = \frac{L_A}{L_B} \frac{R_B^2}{R_A^2}$$

If it's the same luminosity star but at different distances, then you have:

$$2.5^{m_B - m_A} = \frac{R_B^2}{R_A^2}$$

If one of the distances, R_A, is the standard distance of 10 parsecs, then the magnitude at that distance is the absolute magnitude M, and you have:

$$2.5^{m-M} = \frac{R^2}{(10\text{ parsec})^2} \quad \text{or} \quad R = \sqrt{2.5^{m-M}} \times 10\text{ parsecs}$$

Or the more exact version:

$$R = \sqrt{100^{(m-M)/5}} \times 10\text{ parsecs} = 10^{(m-M)/5} \times 10\text{ parsecs}$$

Example using Hertzsprung Russell Diagram

An astronomer sees a star that is very blue, and about 25,000 K. The astronomer is pretty sure it is not a white dwarf.

What is this star's absolute magnitude?

I get about -5. Your mileage my vary.

This star is observed to have an apparent magnitude of +5. How far away is it?

$R = 10^{[5-(-5)]/5} \times 10\text{ parsecs} = 10^{10/5} \times 10\text{ parsecs} = 10^2 \times 10\text{ parsecs} = 1000\text{ parsecs}$

Cute, eh? We got a distance without using geometry.