Problem Set 14 — Goes with Chapter 11

For Quantum Mechanics, Thursday, Apr. 2, 2026

The reading that goes with the problem set is Sections III-11-1, III-11-2, III-11-3. Please also take a look at III-11-4, because it might be nice to discuss that in class. We are putting off III-11-6.

1. How do the Pauli Matrices "Transform"

This problem is based on a statement at the middle of p. 11-4. For at least one Pauli matrix and one transformation, we want to see that the Pauli matrix transforms like a vector. We should do σ_x, and the transformation could be a rotation around the $y-$axis. We expect σ_x to "transform" into some combination of σ_x and σ_z. Now I will try to explain what that might mean. Also, Feynman warned us that this is going to be a lot of algebra. That is why we are only doing one Pauli matrix and just a simple rotation by ϕ about the $y-$axis. Let's get more precise....

Let $|\psi>$ and $|\chi>$ be two states. Let $R_y(\phi)$ be a rotation about the $y-$axis. I am deliberately being a little vague about whether we are rotating the apparatus that measures the states defined in basis S to a new basis T, or whether we are rotating the states themselves, because I am frankly not sure which we mean. Whichever the interpretation is, you can find the effect of $R_y(\phi)$ back in the tables on p. III-6-14.

Now consider $R_y(\phi)\,|\psi>$ and $R_y(\phi)\,|\chi>$. Give them names $|\psi_\phi> \equiv R_y(\phi)\,|\psi>$ and $|\chi_\phi> \equiv R_y(\phi)\,|\chi>$.

What this problem boils down to is calculating $<\chi_\phi\,|\,\sigma_x\,|\,\psi_\phi>$ and hopefully discovering that it is some combination of $<\chi\,|\,\sigma_x\,|\,\psi>$ and $<\chi\,|\,\sigma_z\,|\,\psi>$. The combination has to be independent of what $|\psi>$ and $|\chi>$ are chosen. They could be any states whatsoever!

It might help to use this: $\sigma_x = |S+><S-| \; + \; |S-><S+|$.

Hopefully we'll find a combination of σ_x and σ_y that makes perfect sense as if the σ's were undergoing a vector transformation by ϕ around the $y-$axis. Apply the right-hand rule carefully if you want to get precise about the difference between ϕ and -ϕ or between transforming the states and transforming the apparatus.

2. Feynman Problem 78.1(a)

Just do this problem mechanically. This time we aren't trying to prove they are vectors. We are just discovering some vector-like identities. Only do Part (a). I have no idea what Feynman wants in (b).