Quantum Mechanics — Exam 2

Monday, Mar. 23, 2026

1. Black-body Radiation Formula Core Steps

In this problem, I am just asking you to repeat part of the fundamental derivation from Chapter 4.

(a) Application of Detailed Balance

After doing some already quite tricky arguments about photons as bosons, Feynman deduced that:

(i) If $N_g \bar{n} |a|^2$ is the rate at which photons are absorbed by N_g atoms in the ground state,

(ii) then $N_e(\bar{n}+1) |a|^2$ is the rate at which photons are emitted by N_e atoms in an excited state.

By the Principle of Detailed Balance, these rates must be the same.

Just equate the rates and cancel the unspecified amplitude-squared, $|a|^2$.

(b) Incorporating Statistical Mechanics

A fundamental result from statistical mechanics is that $\frac{N_e}{N_g} = e^{-\Delta E/kT}$. Use this fundamental result and your answer from (a) to get a formula for $\bar{n}$.

(c) Rewriting Energies

The energy difference between states, ΔE, is also the energy of the emitted or absorbed photon, $\hbar\omega$, so substitute that into the formula for $\bar{n}$.

Also, since these $\bar{n}$ photons each have energy $\hbar\omega$, to give the total energy of these $\bar{n}$ photons, just multiply your result for $\bar{n}$ by $\hbar\omega$.

NOTE: There was a lot of work preceding and following these core steps to get the final blackbody radiation formula. I’d encourage you to review it because it is remarkable and widely used. The work preceding involves important Bose statistics facts, and the work following involves the size of the box, and the number of allowed wavelengths in any given frequency interval.

2. Spin-One Stern-Gerlach Apparatuses

Let us just repeat a problem that Feynman gave us. Let 9° just be denoted α. There is no need to plug in the actual value.

72.3 A set of three "improved-type" Stern-Gerlach experiments is set up for spin one particles as shown in as shown in Fig. 72-2. The three apparatuses are placed along a straight line, but the T apparatus is rotated about this line by an angle of 9° with respect to the two S apparatuses. A beam of spin one particles enters from the left. The beam which leaves the first S apparatus has an intensity of N_1 particles per second.

(a) What is N_2, the intensity of the beam leaving the T apparatus?

(b) What is the intensity N_3 of the beam that leaves the last S apparatus?

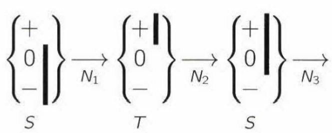


Figure 72-2

You will need entries from this table:

$$
\begin{aligned}
\langle +T \mid +S \rangle &= \tfrac{1}{2}(1+\cos\alpha),\\
\langle 0T \mid +S \rangle &= -\frac{1}{\sqrt{2}}\sin\alpha,\\
\langle -T \mid +S \rangle &= \tfrac{1}{2}(1-\cos\alpha),\\
\langle +T \mid 0S \rangle &= +\frac{1}{\sqrt{2}}\sin\alpha,\\
\langle 0T \mid 0S \rangle &= \cos\alpha,\\
\langle -T \mid 0S \rangle &= -\frac{1}{\sqrt{2}}\sin\alpha,\\
\langle +T \mid -S \rangle &= \tfrac{1}{2}(1-\cos\alpha),\\
\langle 0T \mid -S \rangle &= +\frac{1}{\sqrt{2}}\sin\alpha,\\
\langle -T \mid -S \rangle &= \tfrac{1}{2}(1+\cos\alpha).
\end{aligned}
\tag{5.38}
$$

You do not need the entries with $-\alpha$ filled in instead of α!

Instead, when you need entries like $< -S \mid 0\,T >$, you use the general principle: $< \psi \mid \chi > \, = \, < \chi \mid \psi >^*$. Since all of the table entries are real, that is simple!

3. Spin-1/2 Stern-Gerlach Apparatuses

Chapter 6 was another tour de force. I did not go slowly enough through Feynman’s arguments to follow them. It was just too much. Instead, I assume all of us just skipped straight to his result, which was Tables 6-1 and 6-2, and used those to do the problems. Here is Table 6-2:

Table 6-2

The amplitudes $\langle jT \mid iS \rangle$ for a rotation $R(\phi)$ by the angle ϕ about the z-axis, x-axis, or y-axis

$R_z(\phi)$

$\langle jT \mid iS \rangle$	$+S$	$-S$
$+T$	$e^{i\phi/2}$	0
$-T$	0	$e^{-i\phi/2}$

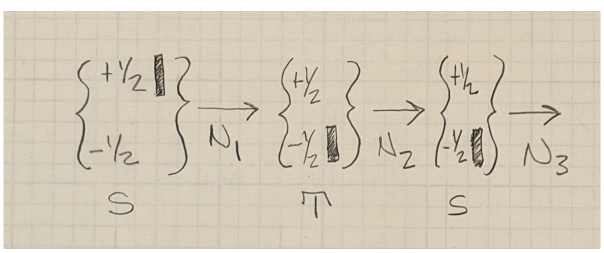
$R_x(\phi)$

$\langle jT \mid iS \rangle$	$+S$	$-S$
$+T$	$\cos\phi/2$	$i\sin\phi/2$
$-T$	$i\sin\phi/2$	$\cos\phi/2$

$R_y(\phi)$

$\langle jT \mid iS \rangle$	$+S$	$-S$
$+T$	$\cos\phi/2$	$\sin\phi/2$
$-T$	$-\sin\phi/2$	$\cos\phi/2$

Here is the new setup:



This time have apparatus T spun by an angle ϕ about the x axis.

First, what do you expect for N_2 and N_3 if ϕ is 0° degrees or 180°?

Now just repeat the parts (a) and (b) that were asked in the previous problem but for this situation. This time, when you need entries like $< +S \mid +T >$, because the entries for $R_x(\phi)$ are complex, you will have to pay more attention to the complex conjugation in $< \psi \mid \chi > = < \chi \mid \psi >^*$.

4. Stationary States of Two-State Systems

As in Problem 1, I will just have you repeat a portion of a fundamental derivation, but this time for the ammonia molecule studied in Chapter 8.

Feynman wrote the general state of the ammonia molecule as follows:

$$|\,\psi\rangle = |\,1\rangle C_1 + |\,2\rangle C_2.$$

Then he argued that C_1 and C_2 obey the following equations:

$$i\hbar\,\frac{dC_1}{dt} = E_0 C_1 - A C_2,$$
$$i\hbar\,\frac{dC_2}{dt} = E_0 C_2 - A C_1.$$

(a) Add and subtract these two equations to get differential equations involving only $C_1 + C_2$ and $C_1 - C_2$. The two equations are independent!

(b) Solve each of these equations. You now know $C_1(t) + C_2(t)$ which will involve an integration constant you can call, a, and $C_1(t) - C_2(t)$ which will involve an integration constant you can call, b.

(c) If the initial condition is $C_1(0) = 1$ and $C_2(0) = 0$, what are a and b?

(d) This will take some work, but the final thing to do is to get some nice expressions for $C_1(t)$ and $C_2(t)$ for the initial conditions in (c).

HINT: The final expression for $C_1(t)$ had two factors, and one of the factors was $\cos\frac{At}{\hbar}$.

Name: ___________________

1. / 3
2. / 4
3. / 4
4. / 4

GRAND TOTAL

/ 15