The Special Theory of Relativity — Syllabus

Final Version: December 14th, 2020

Overview

The fundamental paradox that the speed of light is constant (it is observed to be the same regardless of an observer's velocity) is resolved by Einstein's special theory of relativity. However the resolution brings us to even more puzzling confrontations to our intuition. Among the things we have to accept are the relativity of simultaneity (it is impossible for observers whose frames are in relative motion to simultaneously synchronize their clocks), length contraction (moving objects are shortened in their direction of motion), and time dilation (moving clocks run slow, epitomized in the Twin Paradox). The consequences keep playing out, ultimately leading us to the famous $E = mc^2$ formula.

A one-semester course beginning with the evidence ruling out the ether theory, focusing on all of the consequences of special relativity, and culminating with a very brief introduction to Einstein's even more profound general theory of relativity, leaps the student forward from the usual starting point in science of 17th century mechanics straight into 20th century results, and covers a scientific revolution even more unintuitive than the oft-studied Copernican revolution.

Materials

- *Einstein's Legacy*, Julian Schwinger. From Chapter 1 of this text we become familiar with the historical stage set by Newton and Maxwell, and in particular the evidence that the speed of light depends neither on the motion of the source nor the motion of the observer. We will also study the material capturing the state of affairs immediately prior to Einstein's 1905 paper. At that time an area of great interest was the electric and magnetic fields of the moving electron, and whether the entire existence and mass of the electron could be understood from electromagnetism alone.
- *Spacetime Physics, 2nd Edition*, Edwin F. Taylor and John Archibald Wheeler, Freeman, 1992. This is our principal text. A course based on Taylor and Wheeler is dominated by a truly modern understanding of special relativity, and the development is done in sufficiently advanced terms that the student is prepared

for the general theory of relativity. Taylor and Wheeler's final chapter introduces gravity and curved spacetime.

- As a taste of general relativity, we will start into the authors' companion volume, *Exploring Black Holes: An Introduction to General Relativity*. Our focus will be Chapter 2, Sections 2.4 to 2.11 (in the first edition's chapter numbering). Our endpoint will be to understand the gravitational red shift as it is captured in and predicted from the Schwarzschild metric. We will thereby understand the strange passage of time on Planet Miller (orbiting Gargantua) in *Interstellar.*

Daily Schedule

Detailed daily schedules will be kept retrospectively for both Term 2 and Term 3.

Unit Outline

I. The Stage for Special Relativity

Newtonian Relativity
The Ether and Michelson's Interferometric Tests
The Speed of Light is Constant

II. First Principles and Results of Special Relativity

Measurements of Time and Space
Moving Clocks Run Slow
Reference Frames
No Preferred Frame

III. Lorentz Invariance

Length Contraction
The Lorentz Transformation
===== Term 2/3 break =====
The Twin Paradox

World Lines of Accelerating Particles
Lorentz Transformation of Displacements
Proper Time for Moving and Accelerating Particles
Regions of Spacetime
Invariant Hyperbolae

IV. Momentum and Energy

Momentum Conservation in Collisions
Energy Conservation in Collisions
Lorentz Transformation of Momentum and Energy
Examples: Fission, Fusion, Photon Braking

V. Curved Spacetime

Curved Spacetime is Locally Lorentzian
Mass Causes Curvature
Argument for the Existence of Gravitational Waves
The Euclidean metric in Polar and Spherical Polar Coordinates
The Schwarzschild (Black Hole) metric in Spherical Polar Coordinates

Assignments, Exams, Grading

Two half-period (45-minute) exams each term, each worth 30% of the grade for the term. Homeworks due approximately weekly (about 7 in total per term), for 30% of the grade. 10% participation. Semester (both terms) grades combined equally for a final semester grade.

Attendance and Other Expectations

I strongly encourage 100% attendance. However, two absences will be automatically excused with no penalty. If you must be absent, find a classmate to help you catch up.

For homework, neatness is part of clarity and clarity counts. Turn homework in on 8 1/2 x 11 sized ruled paper, and staple your work. Working in pencil with a straight edge and a large eraser for scrubbing out big mistakes is encouraged. Some people may be able to turn in neat work in pen. If so, feel free.

Work independently except when stuck. Then feel free to ask others (including me) for enough help to get un-stuck and go back to working independently.

Create quality diagrams, define variables that capture the problem statement, work in terms of those variables, and plug in numbers at the end.

The participation grade is based on questions brought to class (that demonstrate both close reading and analysis), and on active participation in the problems we work on in class.

I am counting on you to do most of what is usually accomplished in "lecture" outside of the class. That frees up our class time for discussion and working on problems. I am also counting on you to come to my office (at the West Duplex) for help — hopefully well before you feel overwhelmed.

Schwarzchild