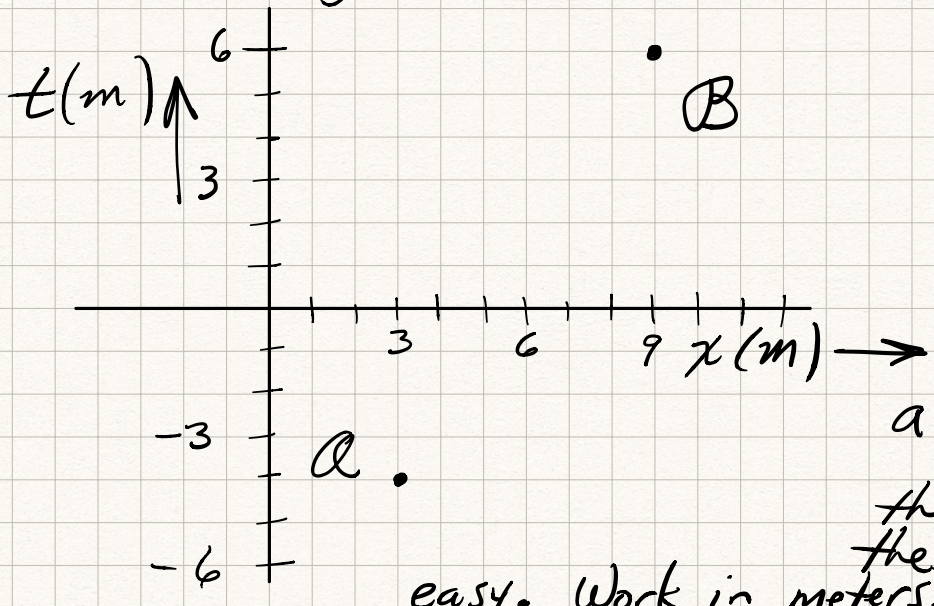
Special Relativity Exam Covering Taylor and Wheeler Chapters 1–3

2020-10-06

1. The following is a spacetime diagram showing two events A and B in the lab frame:



The ticks on the x and t axes represent 1m of space and 1m of time, respectively.

a. What is the interval^2 for the separation between these points. Make life easy. Work in meters. Not meters and seconds.

b. Take the square root of what you got in a, to get the interval. Include units (which are meters) in your answer.

c. Imagine a rocket that passes A just as that event occurs, and B just as that event occurs. What is the elapsed time between A and B <u>in the rocket frame</u>!

d. How fast is this rocket going according to the lab frame? Answer in the natural units for speed, which has no units if you make your life easy. Part d is easy if you look at the spacetime diagram. You don't need parts a, b, or c.

2. The following is the velocity addition formula you have seen and used:

$$v = \frac{v' + v_{rel}}{1 + v' v_{rel}}$$

v = velocity of particle in lab frame

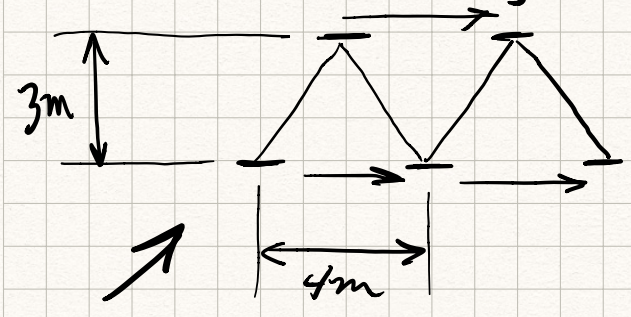
v_{rel} = velocity of rocket in lab frame

v' = velocity of particle in rocket frame

Suppose the particle in the rocket frame is actually a photon, so $V' = 1$.

What is the velocity of the photon in the lab frame?

3. A photon bounces back and forth between two mirrors separated by 3m that are moving rapidly to the right as shown



LAB FRAME DIAGRAM

In one round trip — two round trips are shown at left, but just consider one round trip — the mirrors move 4m to the right.

a. What is the total distance traveled by the photon in one round trip?

It is fine to leave a square root in your answer.

b. How long does the round trip take (in meters)?

c. Knowing that the mirrors traveled 4m in the time you found in b., what must the speed of the mirrors be in the lab frame? It is fine to leave square roots in your answer.

d. How far has the photon gone in one round trip in the frame where the mirrors are at rest?

e. How long does the photon round trip take in the frame where the mirrors are at rest?

4. Doppler Shift

That's $0.1c$, but $c=1$, so just 0.1 is reasonable

A particle going 0.1 emits flashes 1 second apart in its frame. The time dilation formula says these flashes are

$\frac{1}{\sqrt{1-0.1^2}}$ · 1s apart in the lab frame.

a. Using $(1+x)^n \approx 1+nx$ when x is small

first say, what would you use for x and what would you use for n in order to approximate $\frac{1}{\sqrt{1-0.1^2}}$

b. Using these values for n and x what is your approximation for $\frac{1}{\sqrt{1-0.1^2}}$ · 1s ?

c. for definiteness, assume the particle is moving away from you (the observer) not towards you.

How much farther away from you is the particle after each successive flash?

d. The observer measures the reception time between flashes to reach them while they observe from a fixed place. What is the reception time that is measured?

5. A meson lives a time $\tilde{\tau}$ in its own frame. It lives a time t in the lab frame during which it moves a distance vt. This is captured in the following table

	time	distance
lab frame	t	vt
meson frame	$\tilde{\tau}$	0

Find an expression for t (the expression will contain v and $\tilde{\tau}$).