Special Relativity — Term 2 Final Exam Solution

1. Moving Mirrors

Resting distance between mirrors is 3m. Speed $v = \frac{2}{\sqrt{13}}$. Round trip time on last exam was $2\sqrt{13}$ meters of time.

(a) $$v = \frac{2}{\sqrt{13}} \Rightarrow \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-\frac{4}{13}}} = \frac{1}{\sqrt{9/13}} = \frac{\sqrt{13}}{3}$$

So the space between the mirrors has shrunk to

$$\frac{3m}{\sqrt{13}/3} = \frac{9m}{\sqrt{13}} \Longleftarrow$$

By the way, this is about 2.5 meters if you had a calculator or about $\frac{5}{6}$ of the mirror frame length

according to the lab frame observer.

(b) Total distance must equal time

$$\frac{9m}{\sqrt{13}} + t_{right}\frac{2}{\sqrt{13}} \overset{!}{=} t_{right}$$

$$\Longrightarrow \quad t_{right} = \frac{9m/\sqrt{13}}{1-2/\sqrt{13}} = \frac{9m}{\sqrt{13}-2}$$

(c) Total distance must equal time

$$\frac{9m}{\sqrt{13}} - t_{left}\frac{2}{\sqrt{13}} \overset{!}{=} t_{left}$$

$$\Longrightarrow t_{left} = \frac{9m/\sqrt{13}}{1+2/\sqrt{13}} = \frac{9m}{\sqrt{13}+2}$$

(d) $$t_{right} + t_{left} = 9m\left(\frac{1}{\sqrt{13}-2} + \frac{1}{\sqrt{13}+2}\right)$$

(d) (Cont'd)

common denominator

$$t_{right} + t_{left} = 9m \cdot \frac{\sqrt{13}+2 + \sqrt{13}-2}{(\sqrt{13}-2)\cdot(\sqrt{13}+2)}$$

$$= 9m \, \frac{2\sqrt{13}}{13-4} = 9m \, \frac{2\sqrt{13}}{9}$$

$$= 2\sqrt{13} \text{ meters of time}$$

(e) Yes, it must agree, because according to the observer in the mirror frame all that has happened is that the apparatus has been rotated by 90° and that cannot change the round trip time.

It sure looks like a very different experiment according to the lab frame observer, but it must all work out so that the lab frame round trip time is unchanged too, and indeed, it is unchanged:

$2\sqrt{13}$ meters of time

By the way, with a calculator, this is about 7.2m, which is about $\frac{6}{5}$ of the mirror frame round trip of 6m

2. Rocket Approaching Spaceport Runway

(a) Plug $x_{n,m} = nL$ and $t_{n,m} = m\tau$ into the Lorentz transformation

$$t'_{n,m} = \gamma(t_{n,m} - v x_{n,m}) = \gamma(m\tau - vnL)$$
$$x'_{n,m} = \gamma(x_{n,m} - v t_{n,m}) = \gamma(nL - vm\tau)$$

(b) $$x'_{n+1,m} - x'_{n,m} = \gamma((\cancel{n}+1)L - \cancel{vm\tau}) - \gamma(\cancel{nL} - \cancel{vm\tau})$$
$$= \gamma L$$

Some discussion

Gosh, shouldn't this be L/γ not γL?

Well, no! Length contraction is about distance, not coordinate difference. The runway lights are moving in the rocket frame. The coordinate difference has to be done at the same time. We just calculated the coordinate difference at the mth flash of the two lights. The mth flashes do not occur at the same time.

(c) $$t'_{n,m+1} - t'_{n,m} = \gamma((\cancel{m}+1)\tau - \cancel{vnL}) - \gamma(\cancel{m\tau} - \cancel{vnL})$$
$$= \gamma\tau$$

(d) $$x'_{n,m+1} - x'_{n,m} = \gamma(\cancel{nL} - v(\cancel{m}+1)\tau) - \gamma(\cancel{nL} - \cancel{vm\tau})$$
$$= -\gamma v\tau$$

(e) $$\text{interval}^2 = (\gamma\tau)^2 - (-\gamma v\tau)^2$$
$$= \gamma^2(1-v^2)\tau^2 = \tau^2$$

(f) Woo hoo! Yes.

(g) The time difference of the flashes is $\gamma'\tau$. But in that time the flashing light has gotten $v\gamma'\tau$ closer. So the time difference for flash arrivals is

$$\gamma'\tau - v\gamma'\tau = (1-v)\frac{1}{\sqrt{1-v^2}}\tau$$

$$= \frac{\sqrt{1-v}}{\sqrt{1+v}}\tau$$

$$\frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-v}}\cdot\frac{1}{\sqrt{1+v}}$$

(h) Same but the flashing light has gotten $v\gamma'\tau$ farther so the time difference between flash arrivals is

$$\gamma'\tau + v\gamma'\tau = (1+v)\frac{1}{\sqrt{1-v^2}}\tau$$

$$= \frac{\sqrt{1+v}}{\sqrt{1-v}}\tau$$

(i) $t'_{n,0} = -\gamma' v n L$

$$t'_{n+1,0} - t'_{n,0} = -\gamma' v(\cancel{n}+1)L - (\cancel{-\gamma' v n L})$$

$$= -\gamma' v L$$

(j) Runway light $n+1$ had its 0th flash $\gamma' v L$ earlier than runway light n. The rocket and its coordinates have moved $v\cdot\gamma' v L$ in that time.

(k) Correct the $\gamma' L$ we got in (b) by subtracting $v\cdot\gamma' v L$, and we get:

$$\gamma' L - v\cdot\gamma' v L = \gamma'(1-v^2)L = \gamma'\frac{1}{\gamma'^2}L = \frac{L}{\gamma'}$$

Woo hoo! The usual length contraction formula