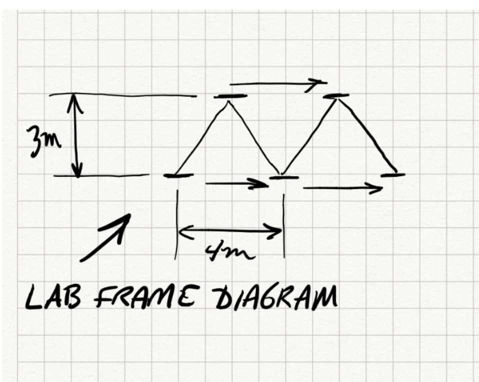
Special Relativity - Term 2 Final Exam

October 16, 2020 — Covering the essential material in Taylor and Wheeler Chs. 1 to 3 and Chapter L

DIRECTIONS: Include units in your answers, but make your life easy!! For the moving mirrors problem you can measure both space and time in meters. Just stick with the units that the problem is given in. To put it differently, for this exam — as in Taylor & Wheeler — c is merely a conversion factor: $1 = c = 3 \times 10^8$ m/s. Since the units of meters and seconds are related, you can work in either of them. There is no need to convert!

1. Moving Mirrors (6 pts)

On the prior exam was a diagram of two mirrors moving to the right. Light was bouncing between them as they moved. On the prior exam you found the speed that the mirrors must be moving, and it was 4 meters / (2 $\sqrt{13}$ meters)= 2/ $\sqrt{13}$.



With those results in mind, here is a new lab frame diagram and a new description:

Instead of having the light bouncing transversely, the mirrors have been rotated by 90° degrees (with no other changes according to an observer in the mirror frame).

For an observer in the lab frame, to make a round trip, first the light moves in the direction of the moving mirrors, and then it moves against the direction of the moving mirrors. The upper arrow shows how much the right mirror has moved to the right during a round trip. The lower arrow shows the left mirror moving the same amount to the right during a round trip.

Two-and-a-half round trips are shown, and the round trips are shown slightly offset for clarity, but this is a 1-dimensional problem. There is no longer any motion in the transverse direction. \

Focus on the first round trip only.

(a) Use the length contraction formula L/γ with $v = 2\big/\sqrt{13}$ to determine how much the 3 meters has shrunk to now that the orientation of the mirrors is in the direction of motion.

(b) Call the time to go from the left mirror to the right mirror t_{right}. Set up an equation that tells you what t_{right} is and solve for t_{right}. (HINT: There is the distance you found in (a). The light has to go that distance plus $t_{\text{right}} \cdot 2\big/\sqrt{13}$.)

(c) Call the time to go from the right mirror to the left mirror t_{left}. Set up an equation that tells you what t_{left} is and solve for t_{left}. (HINT: There is the distance you found in (a). The light has to go that distance minus $t_{\text{left}} \cdot 2\big/\sqrt{13}$.)

(d) Add the resulting expressions for t_{left} and t_{right} that you got in (b) and (c) together and simplify as much as you can to get the total time of the round trip.

(e) Must the round trip time for this new orientation of the mirrors agree with what you got on the last exam, which was $2\sqrt{13}$ meters of time? (Just yes or no is fine.) If it has to agree, but it doesn't, check for errors. If it doesn't have to agree, so be it.

Lorentz Transformation

2. Rocket Approaching Spaceport Runway (9 pts)

This is a 1-dimensional problem, meaning that you do not need any transverse dimensions or coordinates (y, z, y', or z'). In that case, Taylor and Wheeler Equation L-11a is (where as usual, $\gamma \equiv 1\big/\sqrt{1-v^2}$):

$$t' = \gamma(t - v x)$$
$$x' = \gamma(x - v t)$$

Since there is only one velocity in this problem (the rocket velocity), I have just called it v rather than v_{rel}. This is a problem where a rocket is approaching and then flying along a spaceport runway. We definitely **are not** going to worry that the rocket needs to slow down.

Ok, here is the problem:

In the spaceport frame, there is a long set of runway lights at positions $x = nL$, where $n = 0,\ 1,\ 2,\ 3,\ \ldots.$

Also, all the runway lights flash at times $t = \mathrm{m}\tau$ where $m = 0,\ 1,\ 2,\ 3,\ \ldots.$

The spaceport coordinates of flash m of runway light n are therefore

$x_{n,m} = n\,L$
$t_{n,m} = m\,\tau$

(a) Carefully and mechanically plug those facts into L-11a to find expressions for flash m of runway light n in the rocket coordinates. Your answer will look like:

$t_{n,m}{}'$ = an expression involving $v,\ \gamma,\ nL,\ m\tau$
$x_{n,m}{}'$ = another expression involving $v,\ \gamma,\ nL,\ m\tau$

(b) Subtract $x_{n+1,m}{}' - x_{n,m}{}'$ to get the x'-coordinate difference between runway light $n+1$ and runway light n. The difference **will not** be what you expect from the length contraction formula: L/γ.

(c) Subtract $t_{n,m+1}{}' - t_{n,m}{}'$ to get the t'-coordinate difference between runway light n's flash $m+1$ and flash m.

(d) Subtract $x_{n,m+1}{}' - x_{n,m}{}'$ to get the x'-coordinate difference between runway light n's flash m+1 and flash m

(e) Use your answers to (c) and (d) to compute the interval^2 between runway light n's flash $m+1$ and flash m.

(f) You know the answer to (e) better be simple, because interval^2 is invariant and it was simple in the spaceport frame. Is it what you expect? (Just yes or no is fine.)

Parts (g) and (h) are less mechanical. They require the type of reasoning you have been doing in other Doppler shift problems. To simplify to the usual form it will help to use that:

$$\gamma = \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-v}} \cdot \frac{1}{\sqrt{1+v}}$$

(g) Focus on a single runway light n that is still ahead of the rocket as it flies along the runway. What is the time between arrivals **at the rocket front window** of the flashes of this runway light? (HINT: start with your answer to part (c), then subtract off the amount that the rocket moves closer to the light.)

(h) Repeat part (g) for a runway light that the rocket has passed. What is the time between arrivals at the rocket's rear window of the flashes of this runway light?

(i) Go back to your answer for (a). Just put $m = 0$ into that answer and write down:

$t_{n,0}{}' =$

Once again, do a subtraction:

$t_{n+1,0}{}' - t_{n,0}{}' =$

(j) What you got in (i) is a negative number. It says that runway light $n + 1$ had its 0th flash earlier than runway light n. How much did the lights ahead of the rocket get closer to the rocket in this time?

(k) EXTRA CREDIT OF 1 POINT: Can you use this result you got in (j) to reconcile what you got in (b) with the length contraction formula that you know and love?