Special Relativity - Term 3 Exam

November 17, 2020 — Covering the material in Taylor and Wheeler Chs. 4 to 6

1. The Quadruplets Algebraically (4 pts)

Quadruplets A and B are born simultaneously on Earth, and miraculously, quadruplets C and D are also born simultaneously on Proxima b, a planet orbiting Proxima Centauri. Proxima Centauri is a distance d from Earth.

Quadruplet A stays on Earth. Quadruplet B immediately is put on a spaceship and is sent toward Proxima b at speed v.

Quadruplet C stays on Proxima b. Quadruplet D is immediately put on a spaceship and sent toward Earth at speed v (or if you want to be fussy, velocity $-v$, because quadruplet D is moving in the opposite direction of quadruplet B).

Express your answers to all four questions that follow in terms of d, v, and γ, where as usual, $\gamma \equiv \frac{1}{\sqrt{1-v^2}}$.

(a) How old is quadruplet A when quadruplet D arrives at Earth?

(b) How old is quadruplet D when quadruplet D arrives at Earth?

(c) How much older is quadruplet A than quadruplet D when quadruplet D arrives at Earth.

(d) In midflight, a distance $d/2$ from Earth and Proxima b, as the passing quadruplets B and D looked out the window at each other, do they see the same age on each other's watches as their own watches? If not, who is older? If yes, what do the watches read?

2. The Quadruplets Numerically (2 pts)

This is just a plug-in. Re-answer (a)-(d) in the previous problem, but using

$d = 4\,\text{ly}, \quad v = \frac{4}{5}, \quad \gamma =$

Include units. Since $1 = c = \frac{1\,\text{ly}}{\text{yr}}$, you will find it quite convenient to answer all the questions in years.

(a)

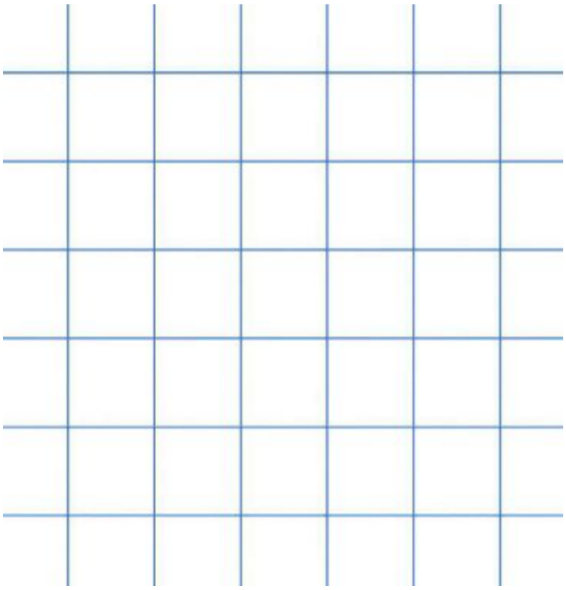
(b)

(c)

(d)

3. A Spacetime Diagram (3 pts)

(a) Using the grid below and a straight edge, draw the spacetime diagram for Problem 2 to scale, with each grid-spacing = 1 ly = 1yr:



Label the worldline of Earth and quadruplet A as “E, A.”
Label the worldline of Proxima b and quadruplet C as “P, C.”
Label the wordlines of quadruplets B and D as “B” and “D,” respectively. Don’t include the quotes :)

3. (CONT'D)

(b) A flash of light is emitted from Earth in the direction of Proxima b at the time that the quadruplets are born. Use a straight edge to draw a faint line at the correct angle, and then trace over your faint line with a wavy line as we do in Feynman diagrams and spacetime diagrams to designate the motion of photons.

(c) If you have done things to scale, you should be able to read off how much ahead of the arrival of quadruplet B at Proxima b the flash arrives. Indicate this interval with its value (which will be in years).

4. Causality and Regions of Spacetime (3 pts)

The following are all assertions about the problem we have been working on. Mostly I am checking that you have internalized Taylor & Wheeler's language. This is important because the language Taylor & Wheeler use is precise and descriptive of spacetime.

Put a T next to each of the assertions that is true and an F next to each assertion that is false.

_____ There is a frame in which the birth of quadruplet B and the arrival of quadruplet B at Proxima b are at the same location.

_____ There is a frame in which the births of quadruplet B and D are at the same location.

_____ The births of quadruplet B and D are spacelike-separated.

_____ The arrival of quadruplet D at Earth is within the forward light cone of the birth of quadruplet D.

_____ The arrival of the flash of light at Proxima b is within the forward light cone of the emission of the flash of light at Earth.

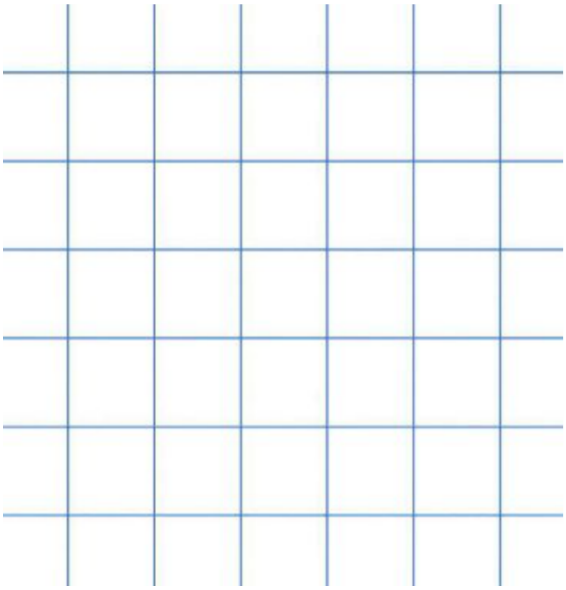
_____ The birth of quadruplet C is outside the light cone of the birth of quadruplet A.

_____ There is a frame where the arrival of the photon at Proxima b and the arrival of quadruplet D at Earth occur at the same time.

_____ The birth of quadruplet B is within the backward light cone of the arrival of quadruplet B at Proxima b.

5. Invariant Hyperbola (3 pts)

(a) Using the grid below, put just the following two events as labeled dots: (1) Event $\mathcal{A}$ will be the birth of quadruplet A, (2) Event $\mathcal{B}$ will be the arrival of quadruplet B at Proxima b.



(b) You should be able to readily calculate the interval between Events $\mathcal{A}$ and $\mathcal{B}$. There is a frame where Event $\mathcal{B}$ occurs at the same place as Event $\mathcal{A}$. Using the calculation of the invariant interval, draw a fainter dot where Event $\mathcal{B}$ would be if you were in this frame.

(c) There is an invariant hyperbola showing all the places where Event $\mathcal{B}$ could be relative to Event $\mathcal{A}$ as viewed from different frames. Sketch in this invariant hyperbola as accurately as you can.