

$$1(a) \quad (\Delta x)^2 < (\Delta t)^2$$

$$\text{or } |\Delta x| < |\Delta t|$$

$$(b) \quad v = \frac{\Delta x}{\Delta t}$$

$$(c) \quad 0 \doteq \Delta t' = \gamma (\Delta t - v \Delta x)$$

$$\Rightarrow \Delta t = v \Delta x$$

$$\Rightarrow v = \frac{\Delta t}{\Delta x}$$

Solution to
term 3

final exam

2. (a) and (b)

	1	2	3
0	$\frac{\text{spacelike}}{\text{no}}$	$\frac{\text{timelike}}{\text{yes}}$	$\frac{\text{spacelike}}{\text{no}}$
1		$\frac{\text{timelike}}{\text{yes}}$	$\frac{\text{lightlike}}{\text{yes}}$
2			$\frac{\text{spacelike}}{\text{no}}$

(c) $\Delta t = 3 \text{ yrs}$ $\Delta x = 1 \text{ ly}$

$$v = \frac{\Delta x}{\Delta t} = \frac{1}{3}$$

(d) $\Delta t = 3 \text{ yrs}$ $\Delta x = 4 \text{ ly}$

$$v = \frac{\Delta t}{\Delta x} = \frac{3}{4}$$

3.

Before

E_A	20
P_A	0
m_A	20

BTW,

$$\sqrt{21} \approx 4.6$$

$$\sqrt{204} \approx 14.3$$

and

$$m_C + m_D = 16.3$$

After

E_C	5
P_C	$-\sqrt{21}$
m_C	2

E_D	15
P_D	$+\sqrt{21}$
m_D	$\sqrt{204}$

$$m_A > m_C + m_D$$

Here is my work for filling in the above tables

$$P_C^2 = E_C^2 - m_C^2 = 5^2 - 2^2 = 21$$

$$P_C = \pm \sqrt{21}$$

$$P_D = \mp \sqrt{21}$$

$$m_D^2 = E_D^2 - P_D^2 = 15^2 - 21 = 225 - 21 = 204$$

$$4(a) \quad E_{\text{electron}} = \sqrt{m_e^2 + p^2}$$

$$(b) \quad E_{\text{positron}} = \sqrt{m_e^2 + p^2}$$

$$(c) \quad E_{\gamma} = 2\sqrt{m_e^2 + p^2}$$

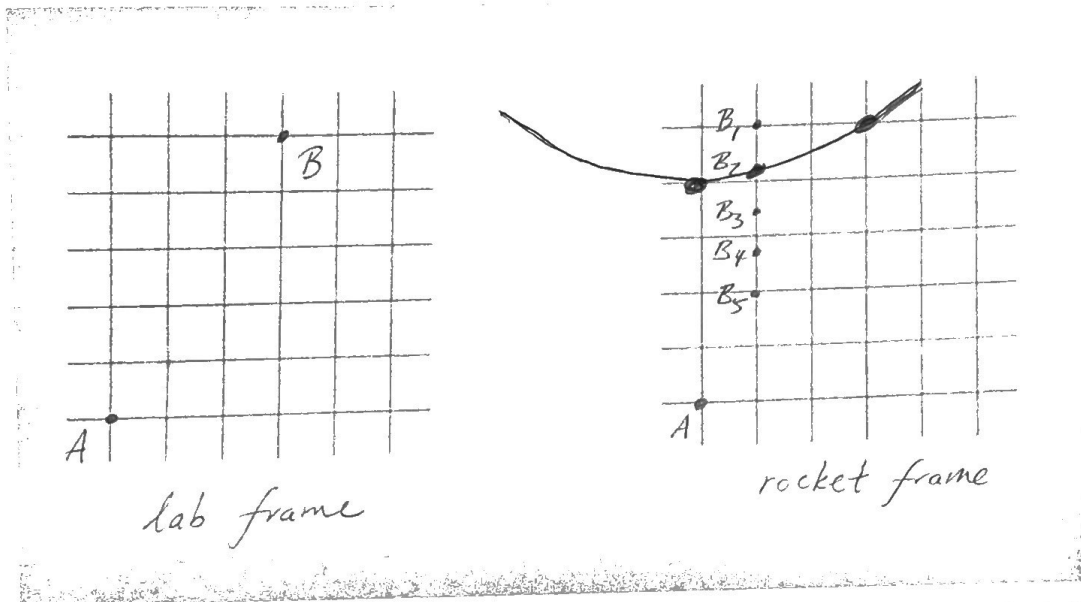
$$(d) \quad p_{\gamma} = 0$$

$$(e) \quad E_{\gamma}^2 - p_{\gamma}^2 = 4(m_e^2 + p^2) > 0$$

but the mass of a photon is zero.

5. Invariant Hyperbola

On the left are two events A and B in the lab frame. On the right are 5 possible positions for B in the rocket frame. On both plots, each grid spacing in the horizontal direction represents one light-year, and each grid spacing in the vertical direction represents one year.



On the right hand plot, sketch the invariant hyperbola of possible positions for event B relative to event A . It will go through only one of the events B_1 to B_5 . HINT: Computing the invariant τ^2 might help you draw an accurate hyperbola.

$$\tau^2 = 5^2 - 3^2 = 16$$

$$\tau = 4$$

Hyperbola goes through B_2