Special Relativity - Term 3 Final Exam

December 8, 2020 — Covering the material in Taylor and Wheeler Chs. 4 to 8

1. Timelike- and Spacelike-Separated Points in Different Frames (4 pts)

In the lab frame, point A has coordinates t_A, x_A and point B has coordinates t_B, x_B. Let's give names to the difference in these coordinates, $\Delta t = t_B - t_A$, and $\Delta x = x_B - x_A$.

(a) In terms of Δt and Δx, write an equation or inequality that is equivalent to saying points A and B are timelike-separated:

(b) Since A and B are timelike-separated, it is supposedly true that there is a rocket frame in which they have the same x – coordinate. What is the velocity, v, of the rocket frame relative to the lab frame for which this is true?

(c) We have shown that Δt and Δx transform the same way as the coordinates. Specifically, we have shown that in a new frame moving with velocity v, $\Delta t'$ and $\Delta x'$ are related to Δt and Δx as:

$\Delta t' = \gamma * (\Delta t - v * \Delta x)$

$\Delta x' = \gamma * (\Delta x - v * \Delta t)$

Suppose A and B are spacelike-separated. What velocity v does the rocket frame need to have so that A and B occur simultaneously (at the same time) in that frame.

2. Analysis of Taylor & Wheeler 6-2

The first table lists the space and time coordinates of three events plus the reference event (event 0) as observed in the laboratory frame.

EXERCISE 6-2

LABORATORY COORDINATES OF THREE EVENTS

	t (years)	x (years)	y (years)
Event 0	0	0	0
Event 1	3	4	0
Event 2	6	5	0
Event 3	8	8	3

(a) In the top half of each box in the table below, write the nature of the interval—timelike, lightlike, or spacelike— between the two corresponding events in the table above.

(b) In the bottom half of each box in the table below, write "yes" if it is possible that one of the events caused the other, and "no" if it is not possible.

EXERCISE 6-2

INTERVAL BETWEEN EVENTS: TIMELIKE, LIGHTLIKE, OR SPACELIKE?

	Event 1	Event 2	Event 3
Event 0			
Event 1	→		
Event 2	→	→	

(c) Find the speed (with respect to the laboratory frame) of a rocket frame in which Events 1 and 2 occur at the same place — or write "impossible" if it is impossible.

(d) Find the speed (with respect to the laboratory frame) of a rocket frame in which Events 0 and 1 occur at the same time — or write "impossible if it is impossible.

3. Do Parts (a) - (f) of Taylor & Wheeler 7-8

NOTE: Taylor and Wheeler do not specify the units, so you can leave them off.

ALSO: It is fine to leave answers with square roots, e.g., $\sqrt{34}$.

7-8 rocket nucleus

A radioactive decay or "inverse collision" is observed in the laboratory frame, as shown in the figure.

Suppose that $m_A = 20$ units, $m_C = 2$ units, and $E_C = 5$ units.

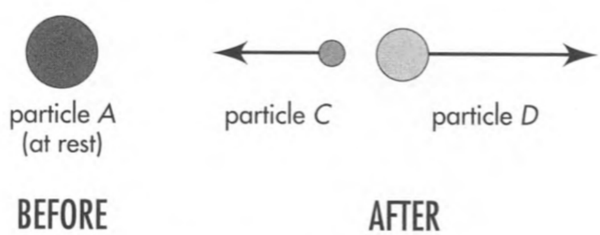
a What is the total energy E_A of particle A?

b From the conservation of energy, find the total energy E_D (rest plus kinetic) of particle D.

c Using the expression $E^2 - p^2 = m^2$ find the momentum p_C of particle C.

d From the conservation of momentum, find the momentum p_D of particle D.

e What is the mass m_D of particle D?



EXERCISE 7-8. *Radioactive decay of a particle.*

f. Only one following is true. Circle the true one:

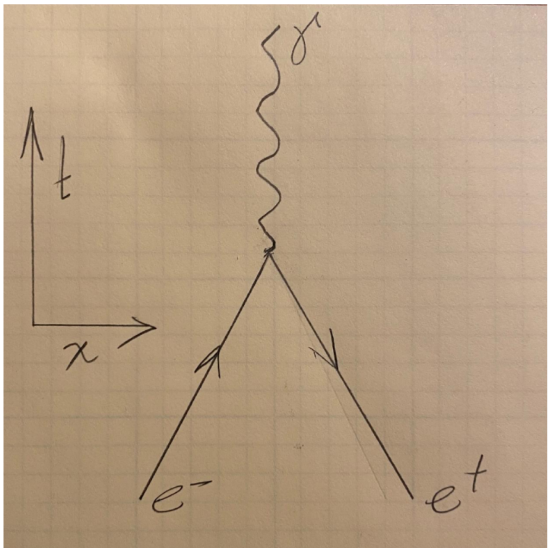
$m_A = m_C + m_D$

$m_A > m_C + m_D$

$m_A < m_C + m_D$

4. Electron and Positron Collide

The simplest Feynman diagram in quantum electrodynamics is an electron and a positron meeting to produce a photon. An electron and a positron have the same mass m_e. Quantum mechanically, an electron and a positron can meet to produce a photon, at least briefly. Classically, it is forbidden because it cannot conserve energy and momentum. You are about to show that it is forbidden. First, here is the diagram:



If time is going upward in this diagram and space is the horizontal direction, then the electron is coming in from the left, the positron is coming in from the right. They meet and the photon is produced, and only the photon continues on after the collision. Here's the trick for your proof: choose a frame where the electron has momentum p and the positron has equal and opposite momentum $-p$. Such a frame always exists for the collision of two particles. It is called the center-of-mass frame.

NOTE: All of your answers for (a) to (d) should be written only in terms of the two variables in the description, namely, p and m_e.

(a) What is the energy of the electron in this frame?

(b) What is the energy of the positron in this frame?

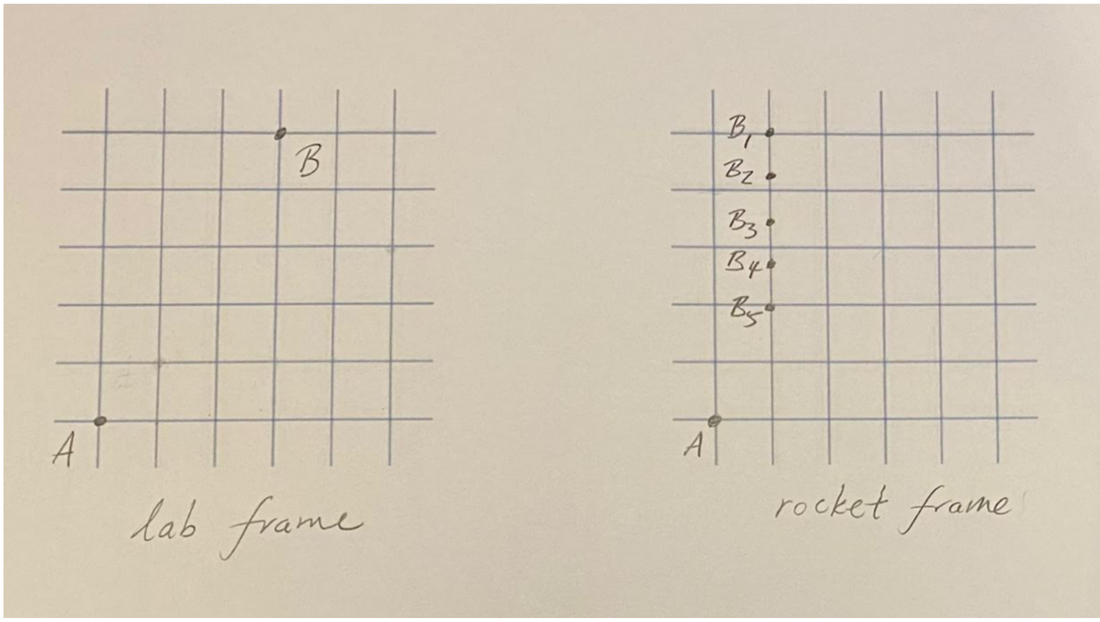
(c) What is the energy of the photon in this frame?

(d) What is the momentum of the photon in this frame?

(e) Why are your answers for (c) and (d) in impossible for a photon?

5. Invariant Hyperbola

On the left are two events A and B in the lab frame. On the right are 5 possible positions for B in the rocket frame. On both plots, each grid spacing in the horizontal direction represents one light-year, and each grid spacing in the vertical direction represents one year.



On the right hand plot, sketch the invariant hyperbola of possible positions for event B relative to event A. It will go through only one of the events B_1 to B_5. HINT: Computing the invariant τ^2 might help you draw an accurate hyperbola.