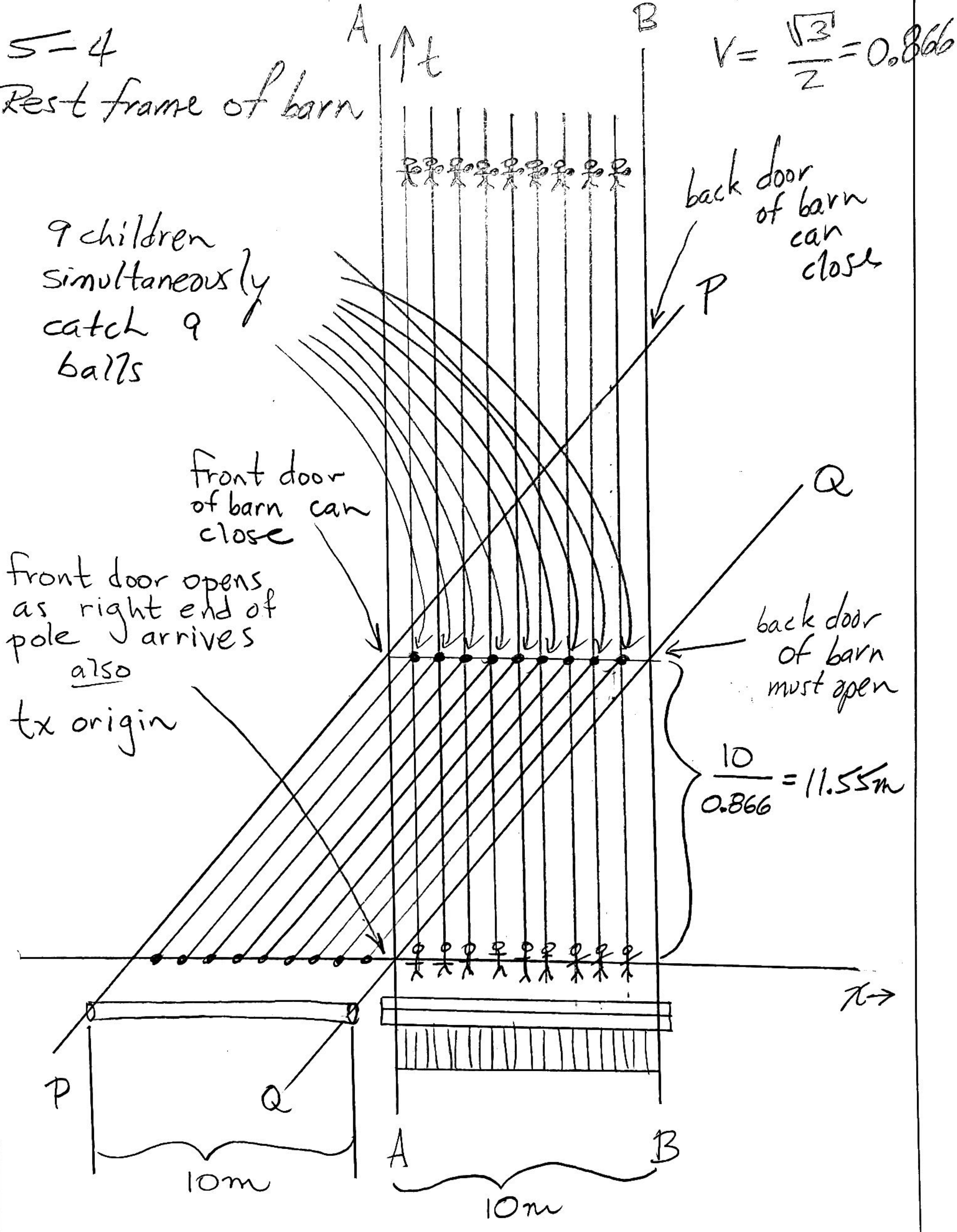
HW 10 Solution

$\gamma = 2$

$V = \frac{\sqrt{3}}{2} = 0.866$

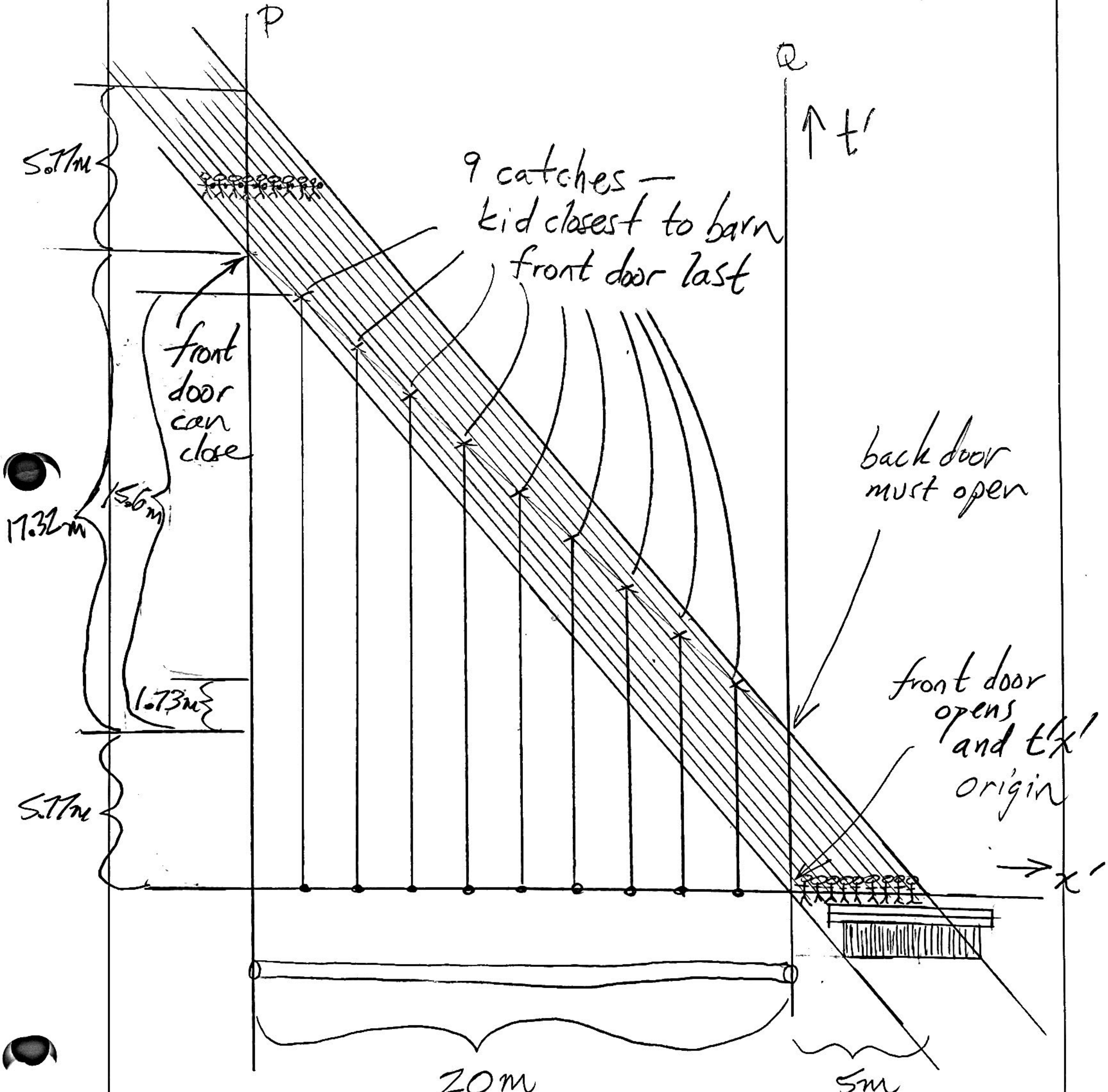
5-4

Rest frame of barn



5-4

Rest frame of pole



Let's describe the scene as we are asked to in 5-4(c)....

I was riding along with the 9th and last tennis ball near the left (-x) end of a 20m pole zipping across the landscape at 0.866. Up ahead a shrunken 5m long barn is approaching, and just as the right (+x) end of the pole arrives at the front door of the barn, the farmer opens it. Inside I see 9 children lined up. We travel right for 5m / 0.866 = 5.77m of time and at that point the right end of the pole reaches the back door of the barn. I am still 15m from the front door which is still open.

Luckily, the back door of the barn is thrown open and the right end of the pole is able to pass through. Very soon after this, the farthest child who was 0.5m towards me from the back door, catches the rightmost ball which was 2m from the end of the pole. As you can see, this required an additional 1.5m of motion which takes 1.5m / 0.866 = 1.73m of time.

Another 1.73m of time elapse and the next child is able to catch the next ball. A total of 9 * 1.5m / 0.866 = 15.6m of time elapses at which point the ball I am riding with is caught by the child closest to the front door. 10 * 1.5m / 0.866 = 17.32m of time after the back door is thrown open, the left end of the pole passes into the barn and the front door shuts. Still another 5m / 0.866 = 5.77m of time elapses and the left edge of the pole passes through the back door and the back door shuts behind it.

Afterwards we interviewed the kids, and they said that the back door opened, they caught all the balls, and the front door closed all at the same dang time. This is nuts because I saw the whole thing unfold and the only way they could claim that they caught the balls at the same time is if the rightmost kid's clock was fast, because that kid made the first catch. Or alternatively, the leftmost kid's clock must have been slow, because that kid caught my ball distinctly after all the others.

This story is told in the rest frame of the pole. The five circled times in the story are shown at the left edge of the drawing on the previous page.

5-5 (a) from $\overline{AC} = \overline{AB}$

$$\Delta t = v\Delta t + \lambda_{reflected}$$

from $\overline{ED} = \overline{EF} = \overline{CF} - \overline{EF}$

$$\Delta t = \lambda_{incident} - v\Delta t$$

(b) Solve

$$\Delta t - v\Delta t = \lambda_{reflected} \qquad \Delta t + v\Delta t = \lambda_{incident}$$

$$(1-v)\Delta t = \lambda_{reflected} \qquad (1+v)\Delta t = \lambda_{incident}$$

$$\Delta t = \frac{\lambda_{reflected}}{1-v}$$

$$(1+v)\frac{\lambda_{reflected}}{1-v} = \lambda_{incident}$$

Use $\lambda_{reflected} = \frac{c}{f_{reflected}}$ and $\lambda_{incident} = \frac{c}{f_{incident}}$

$$\frac{1+v}{1-v}\frac{\cancel{c}}{f_{reflected}} = \frac{\cancel{c}}{f_{incident}}$$

(c's cancel and in any case c=1 in our units)

$$f_{reflected} = \frac{1+v}{1-v} f_{incident}$$

5-5(c) $\frac{1}{1-v} \approx 1+v$ (error is $O(v^2)$)

$$\frac{1+v}{1-v} \approx (1+v)(1+v) = 1+2v+O(v^2)$$

$$f_{reflected} \approx f_{incident}(1+2v)$$

$$\underbrace{f_{reflected} - f_{incident}}_{\text{call this } \Delta f} \approx 2v \underbrace{f_{incident}}_{\text{call this } f}$$

$$\Delta f \approx 2vf \quad \text{or} \quad \frac{\Delta f}{f} \approx 2v$$

5-5(d)

$$\Delta f = 2 \cdot 100\frac{km}{hr} \cdot 10.525 \times 10^9 \, Hz$$

$$= 2 \times \underbrace{\frac{10^5 \cancel{m}}{3600 \cancel{s}}}_{55.55} \underbrace{\frac{1 \cancel{s}}{3 \times 10^8 \cancel{m}} \cdot 10.525 \times 10^9 \, Hz}_{35.08 \, Hz}$$

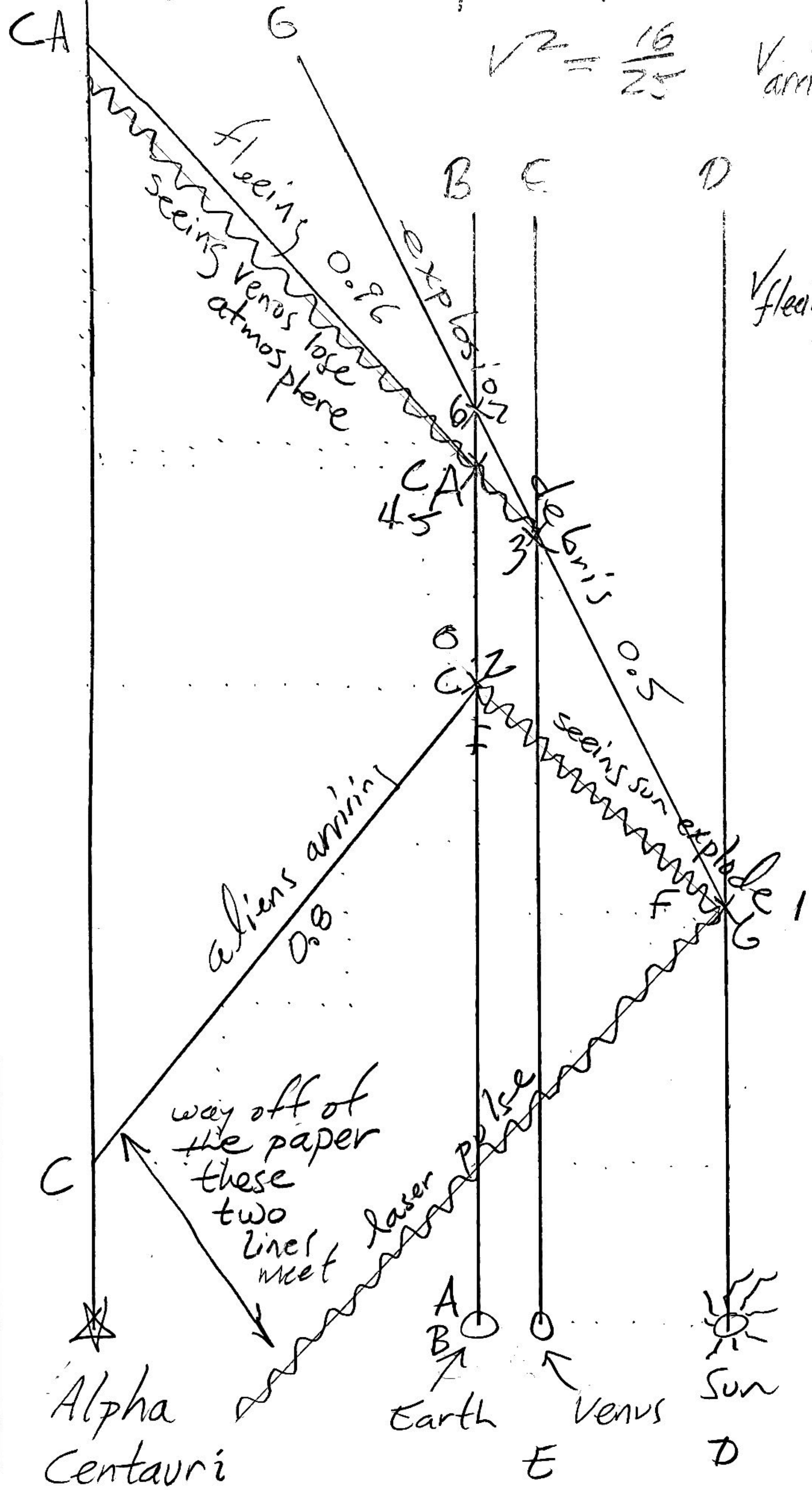
$$= 1949 \, Hz$$

If you want accuracy of 1km/hr, you need 1% of that which is ≈ 20 Hz

$\gamma = \frac{5}{3} \Rightarrow \frac{25}{9} = \frac{1}{1-v^2} \quad \frac{9}{25} = 1-v^2$

$v^2 = \frac{16}{25}$ $\quad v_{arriving} = \frac{4}{5} = 0.8$

arriving



$v_{fleeing} = \frac{24}{25} = 0.96$

fleeing (same calculation as for $v_{arriving}$)

5–6

A summer evening's fantasy

O and Z are coincident

4 and 5 are coincident