Special Relativity HW 11 Solution

Problems 6-2(d), 6-3, and 6-4(b)

6-2(d) We have found that displacements transform the same way as coordinates. If a displacement is Δt, Δx in the lab frame, then in the rocket frame

$$\Delta t' = \gamma'(\Delta t - v\Delta x) \quad \text{and} \quad \Delta x' = \gamma'(\Delta x - v\Delta t)$$

The fact that this looks so much like L-11a is an example of something that happens for any kind of linear transformation. Part (d) asks us to find a frame where event 2 and event 3 occur at the same time. In other words, demand

$$0 \overset{!}{=} \Delta t' = \gamma'(\Delta t - v\Delta x) \implies \Delta t - v\Delta x = 0$$

$$\implies v = \frac{\Delta t}{\Delta x}$$

$$\Delta t = t_3 - t_2 = 8 - 6 = 2\text{yrs}$$
$$\Delta x = x_3 - x_2 = 8 - 5 = 3\text{yrs}$$

$$v = \frac{2\text{yrs}}{3\text{yrs}} = \frac{2}{3}$$

Note that achieving this speed is theoretically possible (although maybe not realistic) because $|\Delta t| < |\Delta x|$ so $|v| < 1$.

6-3 Again we start with

$$\Delta t' = \gamma(\Delta t - v\Delta x) \qquad \Delta x' = \gamma(\Delta x - v\Delta t)$$

For part (a), we are asked to find a frame where $\Delta t' = 0 \Rightarrow \gamma(\Delta t - v\Delta x) = 0$

$$\Rightarrow \Delta t = v\Delta x \Rightarrow v = \frac{\Delta t}{\Delta x}$$

We have found the velocity, and this velocity will be reasonable $-1 < v < 1$ if and only if $|\Delta t| < |\Delta x|$ which is exactly the condition of spacelike separation.

For part (b), demand $\Delta x' = 0 \Rightarrow \gamma(\Delta x - v\Delta t) = 0$ $\Rightarrow \Delta x = v\Delta t \Rightarrow v = \frac{\Delta x}{\Delta t}$. This velocity will be reasonable if and only if $|\Delta x| < |\Delta t|$ which is exactly the condition of timelike separation.

6-4(b) A photon can't really slow down, but through a complex process discussed by the authors, they seem to, so we will pretend that our photon has slowed down to $\frac{1}{n} = \frac{1}{1.00030} = 0.99970$

γ for this speed is $\gamma = \frac{1}{\sqrt{1-(1/n)^2}}$. Rather than just reaching for the calculator, do $\gamma = \underbrace{\frac{1}{\sqrt{1+1/n}}}_{\text{very close to } 1/\sqrt{2}} \underbrace{\frac{1}{\sqrt{1-1/n}}}_{\text{very close to } 1/\sqrt{0.00030}}$

So $\gamma \approx \frac{1}{\sqrt{0.00060}} = \frac{100}{\sqrt{6}} \approx 41$

From the photon's perspective, the distance and time required to zing through 10,000 m of atmosphere at 0.9997 of the speed of light is

$$\frac{h}{\gamma} \approx \frac{10000\text{ m}}{41} = 245\text{ m}$$

(v is so close to 1, that to the precision we are working the distance and time are the same.)