Special Relativity HW 12 Solution

7-7(a), 7-9, 7-10(a) and (b),
8-2, 8-8, and 8-9

7-7(a) How long does a 10^{20} eV proton take to get across the galaxy (according to its time)?

The Milky Way Galaxy is 100,000 light-years across, and if something is going the speed of light, we would say it takes 100,000 years to do the crossing. However, for it, the time is contracted by γ, and we can get γ by

$$\underset{10^{20}\,\mathrm{eV}}{E} = \underset{10^{9}\,\mathrm{eV}}{m}\gamma \implies \gamma = 10^{11}$$

$$\frac{100{,}000\ \mathrm{yrs}}{10^{11}} = 10^{-6}\ \mathrm{yrs} \approx 30\ \mathrm{seconds}$$

7-9 A Sticky Collision

Make some kind of table that allows you to do the accounting. Start with

Before

	m	E	p
A	2	6	$\sqrt{32}$
B	7	9	$-\sqrt{32}$

After

	m	E	p
C	15	15	0

I have filled in what we learned in (a)-(d) in light gray

(a) $E_c = 15$ follows from $E_c = \sqrt{m_c^2 + p_c^2}$

(b) $p_A = \sqrt{6^2 - 2^2} = \sqrt{32}$ $\quad p_B = -\sqrt{32}$

(c) $E_B = 15 - 6 = 9$ $\quad p_B = -\sqrt{32}$

$$m_B = \sqrt{E_B^2 - p_B^2} = \sqrt{81 - 32} = \sqrt{49} = 7$$

(d) Greater, and indeed $15 > 2 + 7$.

T-10 (a) and (b) Colliding Putty Balls

(a) We're going to be doing accounting again

Before

	m	E	p
A	m	$m+K$	
B	m	m	0

After

	m	E	p
C			

The total energy before is $2m+K$, so that tells us the total energy after, $E_C = 2m+K$

(b) $$p_A = \sqrt{E_A^2 - m_A^2} = \sqrt{(m+K)^2 - m^2}$$

$$= \sqrt{2mK + K^2}$$

The total momentum before is just p_A (because B is at rest), and $p_A = \sqrt{2mK+K^2}$, and therefore, $p_C = \sqrt{2mK+K^2}$

I didn't ask you to do it, but now you can finish the last missing accounting entry by computing:

$$m_c = \sqrt{E_c^2 - p_c^2} = \sqrt{(2m+K)^2 - (2mK+K^2)}$$

$$= \sqrt{4m^2 + \underset{2}{\cancel{4}}mK + \cancel{K^2} - 2mK - \cancel{K^2}} = \sqrt{4m^2 + 2mK}$$

8-2 Relativistic Chemistry

(a) 10 metric tons of hydrogen combines with 80 metric tons of oxygen. That's 10,000 times as much as one kilogram combining with eight kilograms, so the energy released will be $10{,}000 \times 10^8$ Joules $= 10^{12}$ Joules

Assuming the energy escaped it must be because $\frac{\Delta E}{c^2}$ of mass disappeared and the amount is

$$\frac{10^{12}\,\text{J}}{(3\times 10^8\ \text{m/s})^2} = \frac{1}{9}\times 10^{-4}\,\text{kg} \approx 10^{-5}\,\text{kg}$$

$$= 10\ \text{mg}$$

(b) $$\frac{10^{-5}\,\text{kg}}{10{,}000\,\text{kg} + 80{,}000\,\text{kg}} = \frac{1}{9}\times 10^{-9} \approx 10^{-10}$$

The sensitivity is still insufficient by a factor of 100.

8-8 First we make the accounting table:

Before

	m	E	p
A	0	E	E
B	m	m	0

After

	m	E	p
C	1.01m	E+m	E

$\sqrt{(E+m)^2 - E^2} = \sqrt{2mE + m^2}$

8-8 (CONT'D)

(a) $1.01m = \sqrt{2mE+m^2}$

Square both sides $(1.01)^2 = 1.0201$

$$1.0201m^2 = 2mE + m^2$$
$$0.0201\, m^{\cancel{2}} = 2\cancel{m}E$$
$$E = 0.01005\, m$$

(b) $E_c = E + m = 1.01005\, m$

E_c is indeed a bit larger than $1.01m$ and $E = 0.01005\, m$ is indeed a bit larger than $0.01\, m$ (the mass increase of the nucleus). That this is so is because C ends up in motion ($p_c = E$ is nonzero).

8-9 Photon "braking"

Before

	m	E	p
A	M	E_A	

$P_A = \sqrt{E_A^2 - M^2}$

After

	m	E	p
D	m	m	0
C	0	E_c	E_c

$$E_c = P_A$$
$$E_A = m + E_c = m + P_A = m + \sqrt{E_A^2 - M^2}$$

$$\Rightarrow E_A - m = \sqrt{E_A^2 - M^2} \Rightarrow \cancel{E_A^2} - 2mE_A + m^2 = \cancel{E_A^2} - M^2$$

$$\Rightarrow 2mE_A = m^2 + M^2 \Rightarrow E_A = (m^2 + M^2)/2m$$