HW 13 Solution

Problem 1

Taylor & Wheeler Problem 8-6

(a) Rest energy is mc^2

While climbing the work you do is

$$\frac{GM_E m}{r^2} \cdot z$$

$GM_E m/r^2$ ← force, z ← height

r ← it doesn't matter whether you put r_E or $r_E + z$ in for r, because $r_E = 6371\,km$ and $z = 300\,m$ less than $\frac{1}{2} \times 10^{-4}$ of r_E

Ratio of work done to rest energy

$$\frac{\frac{GM_E \cancel{m}}{r_E^2} z}{\cancel{m} c^2} = \frac{GM_E}{c^2} \frac{z}{r_E^2} = M_E^* \frac{z}{r^2}$$

$$= 4.44 \times 10^{-3}\,m \frac{300\,m}{(6371\,km)^2}$$

$$= 3.3 \times 10^{-14}$$

Surprisingly small fraction.

Problem 1 (CONT'D)

(b) Rest energy is mc^2

Work done is $\frac{GM_E m}{r_E}$

Ratio is

$$\frac{\frac{GM_E \cancel{m}}{r_E}}{\cancel{m}c^2} = \frac{GM_E}{c^2}\frac{1}{r_E} = M_E^*\frac{1}{r_E}$$

$$= 4.44\times10^3\,m\,\frac{1}{6371\,km}$$

$$= 7.0\times10^{-10}$$

(c) $\frac{\Delta E}{E} = \frac{\Delta f}{f}$ ← because E and f are proportional

Also, $\frac{\Delta E}{E} = -M_E^*\frac{z}{r_E^2} \Rightarrow \frac{\Delta f}{f} = -M_E^*\frac{z}{r^2}$

(d) $\frac{\Delta E}{E} = -M_E^*\frac{z}{r_E} \Rightarrow \frac{\Delta f}{f} = -\frac{M^*_E}{r_E}$

(e) Plug in M^*_E and r_E $\quad \frac{\Delta f}{f} = -7.0\times10^{-10}$

Plug in M^*_{Sun} and r_{Sun} $\quad \frac{\Delta f}{f} = -2.1\times10^{-6}$

Solution to problem 2

In[ ]:= r_{DS} = 6371880
θ_{DS} = N[Pi * (90 - 37.3749) / 180]
ϕ_{DS} = N[Pi * (-117.9802) / 180]

Out[ ]= 6371880

Out[ ]= 0.918481

Out[ ]= -2.05914

DS = "Deep Springs"

In[ ]:= $x_{DS} = r_{DS}$ * Sin[θ_{DS}] Cos[ϕ_{DS}]
$y_{DS} = r_{DS}$ * Sin[θ_{DS}] Sin[ϕ_{DS}]
$z_{DS} = r_{DS}$ * Cos[θ_{DS}]

Out[ ]= -2.37568×10^6

Out[ ]= -4.47172×10^6

Out[ ]= 3.86791×10^6

CM = "Chocolate Mountain"

Rather than using a regular calculator, I used Mathematica as my calculator

In[ ]:= r_{CM} = 6372650
θ_{CM} = N[Pi * (90 - 37.4087) / 180]
ϕ_{CM} = N[Pi * (-117.9285) / 180]

Out[ ]= 6372650

Out[ ]= 0.917891

Out[ ]= -2.05824

In[ ]:= $x_{CM} = r_{CM}$ * Sin[θ_{CM}] Cos[ϕ_{CM}]
$y_{CM} = r_{CM}$ * Sin[θ_{CM}] Sin[ϕ_{CM}]
$z_{CM} = r_{CM}$ * Cos[θ_{CM}]

Out[ ]= -2.37086×10^6

Out[ ]= -4.47239×10^6

Out[ ]= 3.87136×10^6

In[ ]:= d_{linear} = Sqrt[$(x_{CM} - x_{DS})^2 + (y_{CM} - y_{DS})^2 + (z_{CM} - z_{DS})^2$]

Out[ ]= 5966.03

exact, Euclidean expression

In[ ]:= $d_{curvilinear}$ = Sqrt[$(r_{CM} - r_{DS})^2 + r_{DS}^2 (\theta_{CM} - \theta_{DS})^2 + r_{DS}^2$ Sin[θ_{DS}]$^2 (\phi_{CM} - \phi_{DS})^2$]

Out[ ]= 5966.47

approximate, but darned good

Surprisingly close. Within half a meter. $\Delta\theta$ and $\Delta\phi$ are very small.

Problem 3

The circled terms: if $\Delta\phi$ is small

$$\cos\phi_B\cos\phi_A + \sin\phi_B\sin\phi_A$$

$$= \cos(\phi_B - \phi_A) = \cos\Delta\phi \approx 1 - \frac{(\Delta\phi)^2}{2}$$

So we have

$$2r_A^2\left[1 - \sin\theta_B\sin\theta_A\cdot\left(1 - \frac{(\Delta\phi)^2}{2}\right) - \cos\theta_B\cos\theta_A\right]$$

$$= 2r_A^2\left[1 - \sin\theta_B\sin\theta_A - \cos\theta_B\cos\theta_A + \frac{(\Delta\phi)^2}{2}\underbrace{\sin\theta_B}_{\text{can replace with } \sin\theta}\underbrace{\sin\theta_A}_{\text{ditto}}\right]$$

This time the circled terms are

$$\sin\theta_B\sin\theta_A + \cos\theta_B\cos\theta_A = \cos(\theta_B - \theta_A)$$

$$= \cos\Delta\theta \approx 1 - \frac{(\Delta\theta)^2}{2}$$

So we have

$$\cancel{2}r_A^2\left[\cancel{1} + \left(\cancel{1} + \frac{(\Delta\theta)^2}{\cancel{2}}\right) + \frac{(\Delta\phi)^2}{\cancel{2}}\sin^2\theta\right]$$

$$= r_A^2\left[(\Delta\theta)^2 + \sin^2\theta\cdot(\Delta\phi)^2\right]$$

EXACTLY WHAT WE WERE LOOKING FOR