Special Relativity Problem Set 13

1. From Taylor & Wheeler, Chapter 8, do Problem 8-6

HINT/HELP: In part (c), you don't really have to know or use that $E = hf / c^2$ for a photon. You only need to use the fact that E is proportional to f. From this it immediately follows that for a photon,

$$\frac{\Delta E}{E} = \frac{\Delta \mathrm{f}}{f}$$

2. Deep Springs to Chocolate Mountain, an Exercise with Polar Coordinates

a. The coordinates of Deep Springs are $\theta_{\text{latitude}} = 37.3749\,^{\circ}$ and $\phi_{\text{longitude}} = -117.9802^{\circ}$. Our elevation above sea level, is 1580 meters. The Earth's radius at sea level at our latitude is about 6370300 meters.

To complete part (a):

(i) Latitude is conventionally measured from the equator. Take $90\,^{\circ} - \theta_{\text{latitude}}$ to get $\theta_{\text{polar angle}}$. Then convert both the polar angle and the longitude to radians.* Also get r, just by adding 1580 to the sea level. Make sure you have clearly summarized r, θ, and ϕ, to at least six significant figures.

* Why radians? Because the approximations we so often use like $\sin\theta \approx \theta$ for small angles need extra conversion factors if you aren't working in radians.

(ii) Convert r, θ, and ϕ to x, y, and z where:

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

Make sure you have clearly summarized x, y, and z to at least six significant figures.

b. The coordinates of Chocolate Mountain's summit are $\theta_{\text{latitude}} = 37.4087\,^{\circ}$ and $\phi_{\text{longitude}} = -117.9285^{\circ}$. Its summit is 2350 meters above sea level. Repeat (i) and (ii) just like in part (a) to complete this part. As in (a), you need clear summaries of your results so that you can plug in in (c) and (d).

c. Now that you have x, y, and z for both locations, you can take the differences, square them, add them and take the square root to get the distance, as the crow flies, to the summit of Chocolate Mountain:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

Go ahead and do that and find the distance to the nearest 10 meters.

d. You could have done the same thing, at least approximately, without the hassle of converting to x, y, and z, by using our formula for the metric in polar coordinates:

$$d = \sqrt{(\Delta r)^2 + r^2 (\Delta\theta)^2 + r^2 \sin^2\theta\,(\Delta\phi)^2}$$

Where r and θ appear in this formula, it doesn't matter — to the approximation to which we are working — whether you use r_{CM} or r_{DS} for r or whether you use θ_{CM} or θ_{DS} for θ, because they only differ by small amounts, and that difference only alters higher orders in the expansion.

Go ahead and use this formula for d, and once again, you should be able to get the distance to the nearest 10 meters.

If they don't agree (for me they agreed to better than 1 meter), go back and find your mistakes!

3. Polar Coordinates, Trig Identities, and Approximations Practice

Start with

$$x_A = r_A \sin\theta_A \cos\phi_A$$
$$y_A = r_A \sin\theta_A \sin\phi_A$$
$$z_A = r_A \cos\theta_A$$

and

$$x_B = r_B \sin\theta_B \cos\phi_B$$
$$y_B = r_B \sin\theta_B \sin\phi_B$$
$$z_B = r_B \cos\theta_B$$

(CONT'D)

3. (CONT'D)

Define all the following things:

$\Delta x \equiv x_B - x_A$
$\Delta y \equiv y_B - y_A$
$\Delta z \equiv z_B - z_A$

$r_B \equiv r_A + \Delta r$
$\theta_B \equiv \theta_A + \Delta\theta$
$\phi_B \equiv \phi_A + \Delta\phi$

Now get ready with a lot of clean paper and use everything above to simplify

$d^2 = (\Delta\mathrm{x})^2 + (\Delta\mathrm{y})^2 + (\Delta\mathrm{z})^2$

WHEN YOU ARE DONE, you will have derived

$d^2 = (\Delta\mathrm{r})^2 + r^2(\Delta\theta)^2 + r^2 \sin^2\theta\,(\Delta\phi)^2$

ALONG THE WAY, you will need

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\sin(\alpha + \beta) = \cos\alpha\sin\beta + \sin\alpha\cos\beta$

and their trivially related friends (cos is even and sin is odd)

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

and you will need

$\cos\alpha \approx 1 - \frac{\alpha^2}{2}$ + terms of order α^4 if α is small
$\sin\alpha \approx \alpha$ + terms of order α^3 if α is small

Finally, you will need to NEGLECT anything that has the product of three or more small quantities. You are only working to the approximation where you keep small quantities squared.