Second Special Relativity Homework

1. A new version of 1.12

a. $\tau = 18\,ms.$

$t = 180\,ms.$

What is v?

	time	distance
lab frame	t	vt
meson frame	τ	0

b. $t^2 - v^2t^2 = \text{interval}^2$

$\tau^2 - 0^2 = \text{interval}^2$

$t^2 - v^2t^2 = \tau^2$

c. $v^2 = \dfrac{t^2 - \tau^2}{t^2} = 1 - \dfrac{\tau^2}{t^2}$

d. In our problem $\dfrac{\tau}{t} = \dfrac{18ms}{180ms} = \dfrac{1}{10} \Rightarrow \dfrac{\tau^2}{t^2} = \dfrac{1}{100}$

$\Rightarrow v^2 = 0.99 \Rightarrow v = 0.995$

2. 2-8 (a)

$$g_0 + \Delta g = GM\frac{1}{(r_0+\Delta r)^2} = GM\left[r_0^{-2} - 2r_0^{-3}\Delta r + \dots\right]$$

(… neglect)

$$= \frac{GM}{r_0^2}\left(1 - 2\frac{\Delta r}{r_0}\right) = g_0\left(1 - \frac{2\Delta r}{r_0}\right)$$

$$\Rightarrow \Delta g = -2g_0\frac{\Delta r}{r_0}$$

(b) $\Delta y = \frac{1}{2}\Delta g\, t^2 = \frac{1}{2}\left(-2g_0\frac{\Delta r}{r_0}\right)t^2$

$= -g_0\frac{\Delta r}{r_0}t^2$ ← the minus sign is saying that the one at the higher height falls slower. In other words, the two balls are getting farther apart

(c) put in $g_0 = 9.8\ \frac{m}{s^2}$

$r_0 = 6.37\times10^6\ m$

$\Delta r = 20\ m$

$t = 8\ s$

$$|\Delta y| = 9.8\ \frac{20}{6.37\times10^6}\ 8^2\ m = 2000\times10^{-6}\ m = 2mm$$

3. 2-10 a only

$$F = \frac{GMm}{r^2} \qquad F = ma$$

$$\Rightarrow \frac{GMm}{r^2} = ma \Rightarrow a = \frac{GM}{r^2}$$

$$d = \frac{1}{2}at^2 = \frac{1}{2}\frac{GM}{r^2}t^2$$

Plug in $G = 6.67\times10^{-11}\ Nm^2/kg^2$

$M = 10\ kg$

$r = 0.1\ m$

$t = 180\ s$

$$d = \frac{1}{2}\ \frac{6.67\times10^{-11}\cdot10}{0.1^2}\ 180^2\ m = 108{,}000\times10^{-8}\ m$$

$$= 1.08\times10^{-3}\ m \approx 1\ mm$$

4. 2-12 a and b only

(a) $d_{gold} - d_{copper} = \frac{1}{2} g_{gold} t^2 - \frac{1}{2} g_{copper} t^2$

$= \frac{1}{2} \Delta g t^2$

$d = \frac{1}{2} g t^2$

$\Rightarrow \frac{1}{2} t^2 = \frac{d}{g}$

$d_{gold} - d_{copper} = \frac{d}{g} \Delta a =$

$\frac{\Delta d}{d} = \frac{\Delta g}{g}$ $\qquad \frac{\Delta d}{d} < \frac{7cm}{46m} = 0.0015$

$\frac{\Delta g}{g} < 0.0015$

(b) $\frac{\Delta g}{g} < 3 \times 10^{-11}$

$\Delta d < 3 \times 10^{-11} \times 46m = 138 \times 10^{-11} m$

$= 1.38 \times 10^{-9} m = 1.38\ nm$

1.38nm in 46m. How far to get 1mm?

$\frac{1mm}{1.38nm} \times 46m = \frac{10^{-3}}{1.38 \times 10^{-9}} 46m$

$= 33 \times 10^6 m = 33000\ km$