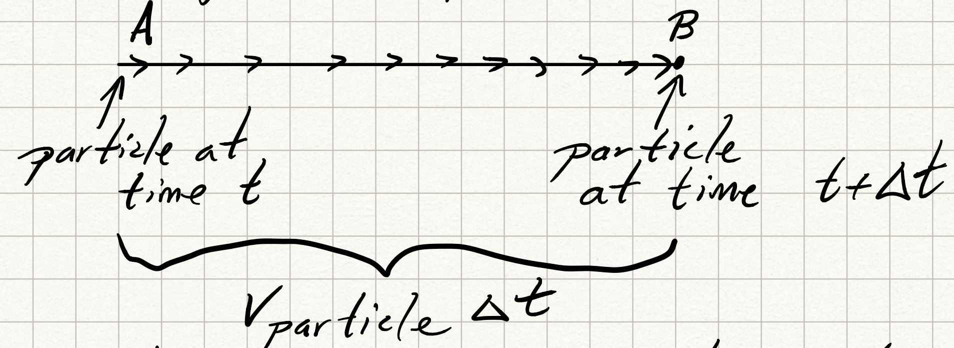
Third SR Homework - Solutions

Problems 3-8, 3-9 and 3-10

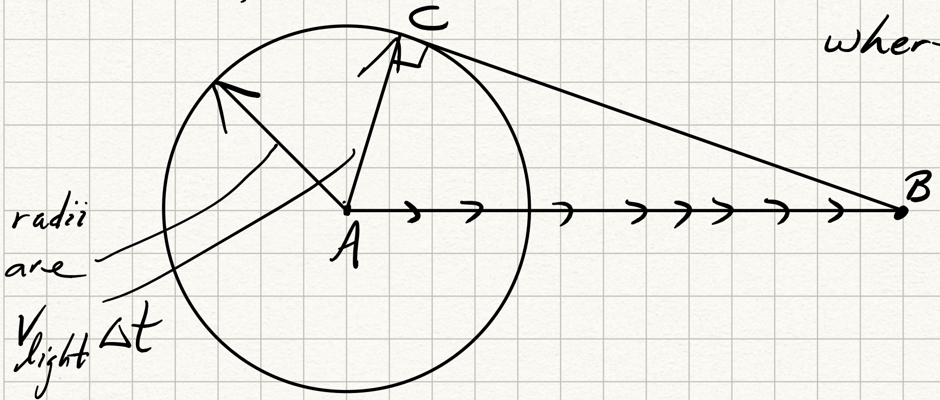
Problem 3-8 Čerenkov Radiation

(a)

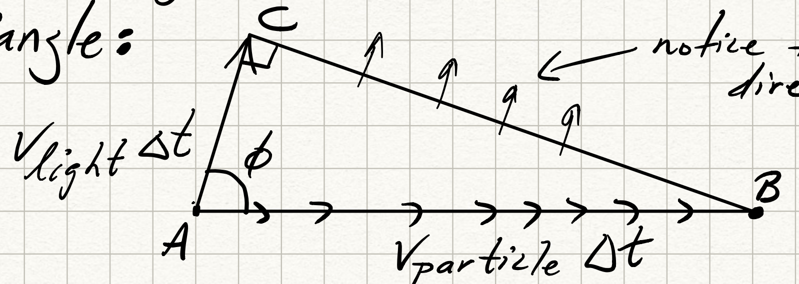
First, draw just the particle motion without the light:



Next, add a circle with radius $V_{light}\Delta t$ representing where the light emitted at time t has gotten to at time $t+\Delta t$



Then add the tangent to the circle BC and a radius (that is perpendicular to the tangent) AC. Then focus on just the triangle:



notice that ϕ defines the direction the wavefront is going — not, what I might have chosen to call the "half-angle of the light cone."

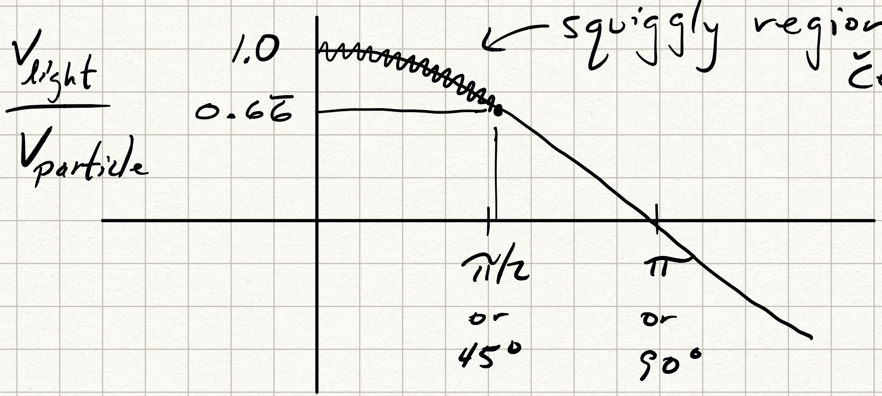
Since it is a right triangle with $A = V_{light}\Delta t$ (adjacent) and $H = V_{particle}\Delta t$ (hypotenuse), we have

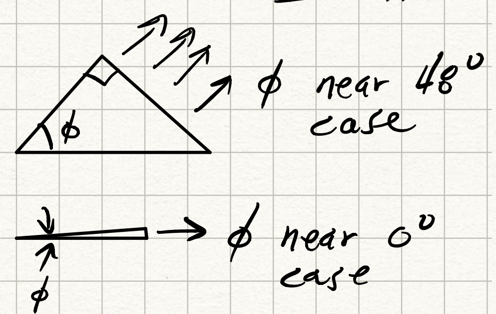
$$\cos\phi = \frac{A}{H} = \frac{V_{light}\Delta t}{V_{particle}\Delta t} = \frac{V_{light}}{V_{particle}}$$

(b) In Lucite, $v_{light} = \frac{2}{3} = \frac{2}{3}c = \frac{2}{3} \cdot 3\times10^8 \frac{m}{s}$
$= 2\times10^8 \frac{m}{s}$

To have Čerenkov radiation, the particle must be going faster than v_{light}. The absolute maximum speed that any particle can go is (anything just less than $c = 3\times10^8 \frac{m}{s}$. Using the limiting case of c itself (even though it can never be attained) for $v_{particle}$, we have

$$\cos\phi = \frac{v_{light}}{c} = \frac{2}{3} \qquad \phi = \cos^{-1}\frac{2}{3} = 48^\circ$$



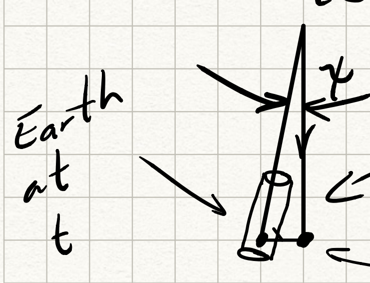


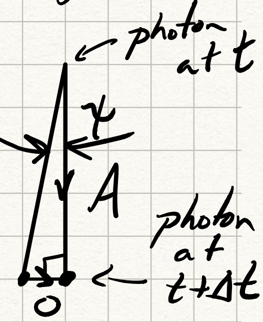
(c) In water $v_{light} = 0.75 = \frac{3}{4} = \cos\phi$

$$\phi = \cos^{-1}\frac{3}{4} = 41^\circ$$

3-9 Aberration of Starlight

(a) It seems we should analyze in sun frame to get ψ?



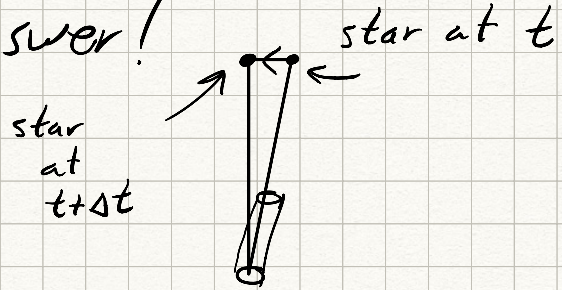


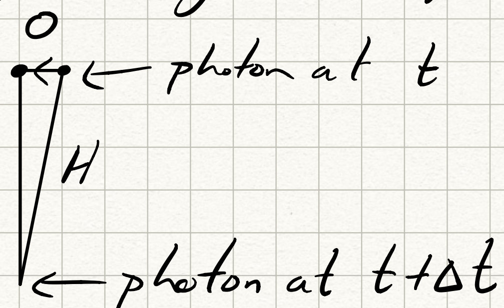
3-9 (a) (CONT'D) $\quad O = V_{earth}\,\Delta t \qquad A = V_{photon}\,\Delta t = c\Delta t$

$$\tan\psi = \frac{O}{A} = \frac{V_{earth}\,\Delta t}{c\Delta t} = \frac{V_{earth}}{c} = V_{earth} \quad \leftarrow \text{in units where } c=1$$

We have found the direction the telescope must point — <u>in the Sun's frame</u>!

In the Earth's frame, we get a different answer!





$$O = V_{earth}\,\Delta t \qquad H = c\Delta t$$

$$\sin\psi = \frac{O}{H} = \frac{V_{earth}\,\Delta t}{c\Delta t} = \frac{V_{earth}}{c} = V_{earth}$$

in units where $c=1$

Don't we want to know the angle of the telescope <u>in our frame</u>!

So the second formula is the one we want!

(b) Plug in numbers

$$\psi = \sin^{-1} V_{earth} \qquad V_{earth} = \frac{30\,\text{km/s}}{3\times10^{8}\,\text{m/s}}$$

$$= \sin^{-1} 10^{-4} \qquad = 10^{-4}$$

The sin of an extremely small angle is just the angle <u>in radians</u>! So

$\psi = 10^{-4}$ radians

3-9 (b) (CONT'D)

multiply by $\frac{180}{\pi}$

Convert to degrees, get $\psi = 0.0057$ degrees

Convert to minutes of arc $\psi = 0.34$ minutes of arc

multiply by 60

Convert to seconds of arc $\psi = 21$ seconds of arc

(c) Compare Sun-frame formula with

naive, not the angle we measure

with Earth-frame formula

$$\psi = \tan^{-1} 0.001 = 0.005729577932^\circ$$

$$\psi = \sin^{-1} 0.001 = 0.005729577960^\circ$$

my calculator displays 10 sig figs

only last two differ

The absolute best angular measurement are made by the Gaia satellite which is the successor to the Hipsarcos satellite. It can do angles between stars to about 10^{-11} radians. That's 0.000000000600°. Not good enough for two reasons: (1) it's 20x as big as the effect we are looking for, and (2) that's it's best measurement of the angles between stars As far as I know, it cannot measure absolute angles with this precision.

(d) Re-evaluate with $V_{rocket} = 0.5$

$$\psi = \tan^{-1} 0.5 = 26.56^\circ \leftarrow \text{Sun-frame}$$

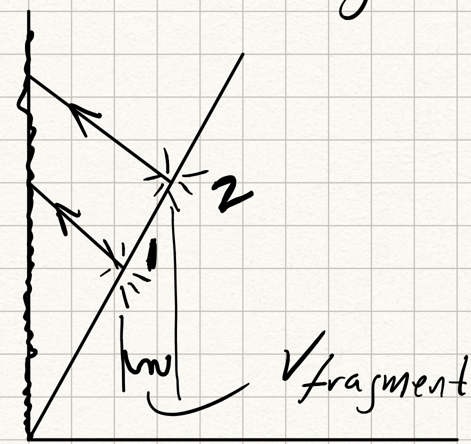
$$\psi = \sin^{-1} 0.5 = 30^\circ \leftarrow \text{Rocket frame}$$

3-10 (a) Expanding universe

$\Delta\tau$ = proper time between flashes (time in frame of emitter)

$\Delta t_{reception}$ = time between flashes (as observed by receiver)

Space-time diagram in receiver's frame:



squiggly line is world-line of receiver

$$\Delta t = \frac{1}{\sqrt{1-V_{fragment}^2}}\,\Delta\tau$$

we don't have to re-derive this.

The second photon has to go $V_{fragment}\,\Delta t$ further.

So it is received not just Δt later, but

$\Delta t + V_{fragment}\,\Delta t$ later

Now rewrite this in terms of $\Delta\tau$

$$\Delta t_{reception} = (1+V_{fragment})\frac{1}{\sqrt{1-V_{fragment}^2}}\,\Delta\tau$$

factor

$$= (1+V_{fragment})\frac{1}{\sqrt{(1+V_{fragment})(1-V_{fragment})}}\,\Delta\tau$$

$$= \sqrt{\frac{1+V_{fragment}}{1-V_{fragment}}}\,\Delta\tau$$

I will discuss the discussion part of 3-9 (a) in class.

3-9 (b) I will also discuss this discussion question in class

(c) Quasars have been seen with $\frac{\Delta t_{reception}}{\Delta \tau} = 5.9$
What is V_{quasar}?

Call the ratio y. Solve

$$y = \sqrt{\frac{1 + V_{fragment}}{1 - V_{fragment}}}$$

for $V_{fragment}$ by first squaring each side.

$$y^2 = \frac{1 + V_{fragment}}{1 - V_{fragment}} \Rightarrow y^2 (1 - V_{fragment}) = 1 + V_{fragment}$$

$$\Rightarrow y^2 - 1 = (y^2 + 1) V_{fragment}$$

$$\Rightarrow V_{fragment} = \frac{y^2 - 1}{y^2 + 1} = 0.944$$

Almost 95% the speed of light.