Special Relativity Homework 6 Solution

L-7 Transformation of y-velocity

This chapter is all about mechanistic application of the Lorentz transformation. The process begins by finding the coordinates in one system....

For this problem, make life easy by choosing the initial position of the particle to be $x'=0$ and $y'=0$

at $t'=0$

A time $\Delta t'$ later it is at $x'=0$ and $y'=v_y'\Delta t'$

The mechanistic process continues by equation L-10 (or sometimes its inverse, equation L-11, if you started with lab frame coordinates)....

Of course the initial position

$t'=x'=y'=0$ transforms into $t=x=y=0$

Later, the particle is at

$t'=\Delta t'$ $x'=0$ and $y'=v_y'\Delta t'$

Find these coordinates in the lab frame:

$$t = v_{rel}\gamma x' + \gamma t' = v_{rel}\gamma \cdot 0 + \gamma \Delta t' = \gamma \Delta t'$$

$$x = \gamma x' + v_{rel}\gamma t' = \gamma \cdot 0 + v_{rel}\gamma \Delta t' = v_{rel}\gamma\Delta t'$$

$$y = y' = v_y'\Delta t'$$

✓✓

Now let's compute $v_x = \frac{\Delta x}{\Delta t} = \frac{v_{rel}\gamma\Delta t' - 0}{\gamma\Delta t' - 0} = v_{rel}$

and $v_y = \frac{\Delta y}{\Delta t} = \frac{v_y'\Delta t' - 0}{\gamma\Delta t' - 0} = \frac{v_y'}{\gamma} = v_y'(1-v_{rel}^2)^{1/2}$ ✓✓

L-10 The tilted meter stick

a. I'm never fond of writing out the "explain why" problems. They are more to encourage you to think and articulate, and it probably doesn't do much if I do the articulating. Here's my attempt:

If a train is moving to the right in the lab frame and lightning strikes both ends at the same time (according to an observer in the lab frame) the person on the train will say the left lightning strike happened later. Or alternatively if the lightning strike occured at the same time in the train frame, then the person in the lab frame will say that the right strike happened later.

The case at hand has the two ends of the meter stick moving upward and at the same height in the lab frame. The person on the says the person in the lab only thinks the left edge of the stick is as high as the right because you keep measuring its height at a later time. The person on the says that if you measure the two ends at the same t' you would see that the right end is higher.

To summarize (perhaps unhelpfully) the person on the rocket says that the further left you go in the lab frame, the more the lab frame clocks are behind.

b. I'm solving this as suggested by the "Discussion" in (b) which is basically a big hint on how to leverage L-7 to get the answer.

The right edge of the meter stick crosses the axis at $t=0 \quad x=\frac{L}{2} \quad y=0$

This transforms to (mechanistically using L-11a)

$$t' = -v_{rel}\gamma\frac{L}{2} \quad x' = \gamma\frac{L}{2} \quad y'=0$$

In other words, it occured before $t'=0$. How much before? $\Delta t' = v_{rel}\gamma \frac{L}{2}$

Meanwhile, from L-7, we know that the center of the stick which will cross at $t'=0$ and is rising is behind by

$$\Delta y' = v_y' \cdot \Delta t' = \gamma v_y \cdot v_{rel} \gamma \frac{L}{2} \qquad \left(\text{also } \Delta x' = \gamma \frac{L}{2}\right)$$

We now have an easy expression for ϕ':

$$\tan\phi' = \frac{\Delta y'}{\Delta x'} = \frac{\gamma^{\cancel{2}} v_y v_{rel} \cancel{\frac{L}{2}}}{\cancel{\gamma} \cancel{\frac{L}{2}}} = \gamma v_y v_{rel}$$

Or perhaps you prefer $\tan\phi' = v_y' v_{rel}$

I have an entire 2nd solution to L-10 that does not leverage L-7:

	Center of stick	Right end of stick
	$t,\ x=0,\ y=v_y t$	$t,\ x=\frac{L}{2},\ y=v_y t$
Use L-11a	$t'=\gamma t,\ x'=-v_{rel}\gamma t,$ $y'=v_y t$	$t'=-v_{rel}\gamma\frac{L}{2}+\gamma t,$ $x'=\gamma\frac{L}{2}-v_{rel}\gamma t$ $y'=v_y t$
get rid of t	$t',\ x'=-v_{rel}t',$ $y'=\frac{v_y t'}{\gamma}$	$\gamma t = t' + v_{rel}\gamma\frac{L}{2}$ $t',\ x'=\gamma\frac{L}{2}-v_{rel}\left(t'+v_{rel}\gamma\frac{L}{2}\right),$ $y'=\frac{v_y}{\gamma}\left(t'+v_{rel}\gamma\frac{L}{2}\right)$

Subtract center coords from right coords

$\Delta x' = \gamma\frac{L}{2}(1-v_{rel}^2) = \frac{1}{\gamma}\frac{L}{2}$, $\Delta y' = v_y v_{rel}\frac{L}{2}$ ← all the t' stuff cancels!

$$\tan\phi' = \frac{\Delta y'}{\Delta x'} = \frac{v_y v_{rel} \cancel{\frac{L}{2}}}{\frac{1}{\gamma}\cancel{\frac{L}{2}}} = \gamma v_y v_{rel}$$

✓✓ As it must if the right end is a fixed amount higher!