Special Relativity HW 7
Taylor & Wheeler 4-1 (a)-(f)

Let's work symbolically and plug in the numbers after simpulifying.

$t_0 = 2200$ A.D. $\qquad c = 1 = \dfrac{1 \text{ light-year}}{\text{year}}$

$v = 0.75$

$D = 8.7$ light-years $= 8.7$ years

$T =$ layover time $= 7$ years

(a) In the Earth frame, at what time does the rocket arrive at Sirius?

$$t_1 = t_0 + \frac{D}{v} = 2200 \text{ A.D.} + \underbrace{\frac{8.7 \text{ years}}{0.75}}_{\frac{4}{3}8.7 = 11.6 \text{ years}}$$

$$= 2211.6 \text{ A.D.}$$

I'm just going to measure all times from 2200 A.D.

$\Rightarrow \quad t_1 = 11.6$ years

(b) $t_2 = t_1 + T = 11.6 + 7 = 18.6$ years

(c) It takes 11.6 years to return just as in (a).

$$t_3 = 18.6 \text{ years} + 11.6 \text{ years} = 30.2 \text{ years}$$

(d) He goes 0.75 of the speed of light, but for James the distance (and time) are Lorentz contracted by γ

$$t_{1 \text{ for James}} = \frac{t_1}{\gamma} = t_1\sqrt{1-v^2} = 11.6 \text{ years} \cdot \sqrt{1-\tfrac{9}{16}}$$

$$= 11.6 \text{ years} \cdot \frac{\sqrt{7}}{4} = 2.9 \text{ years} \cdot \sqrt{7} \approx 7.7 \text{ years}$$

4-1(e) The layover time is T in either frame, because the layover is at rest in the Earth frame, so

$$t_{2 \text{ for James}} = t_{1 \text{ for James}} + T = 7.7 \text{ years} + 7 \text{ years}$$
$$= 14.7 \text{ years}$$

(f) The added time for this part is the same as was calculated for part (d), so

$$t_{3 \text{ for James}} = t_{2 \text{ for James}} + t_{1 \text{ for James}}$$
$$= 14.7 \text{ years} + 7.7 \text{ years} = 22.4 \text{ years}$$

Here is (g), (h), and (i) as well

(g) $$D_{\text{for James}} = \frac{D}{\gamma} = 8.7 \text{ light-years} \frac{\sqrt{7}}{4}$$
$$\approx 5.8 \text{ light-years}$$

that's $\frac{1}{\gamma} = \sqrt{1-v^2}$

(h) That's $t_{1 \text{ for James}} = 7.7$ years and we previously got that Earth clocks read $t_1 = 11.6$ years when James gets to Sirius

(i) That's $t_{2 \text{ for James}} = 14.7$ years and $t_2 = 18.6$ years

The authors recommend repeating these quite straightforward calculations with various other values.