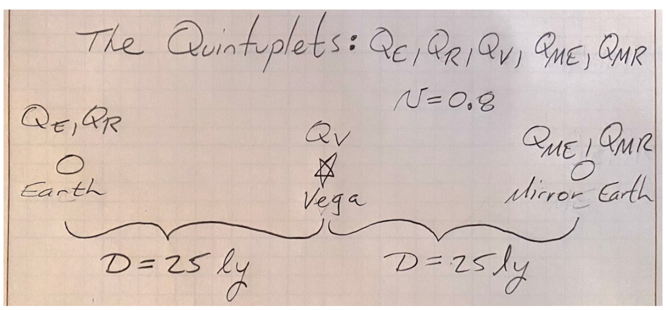
Problem Set 8

Problem 4-1 with New Numbers

Redo 4-1 (a)-(f) with the good old 5/12/13 combination. (I'm alluding to a common example of a triangle with integer sides (some others being 3/4/5 and 7/24/25.) So we are going to choose a star 12 light-years away, and go there at 12/13 of the speed of light, and therefore get there in 13 years, according to earth observers. Since adding layover time isn't particularly interesting, just have the rocketeer return immediately.

A New Problem Inspired by 4-2

Inspired by the discussion of 4-2 in which I invoked triplets, I'm going for quintuplets!!!!! Vega is 25 light-years from Earth. Imagine also, that there is a "Mirror Earth" 25 light-years the other side of Vega. Here is the picture as the quintuplets are simultaneously born.



Of course, "simultaneous" is relative. For this problem, by "simultaneous," I mean "simultaneous according to a system of rods and clocks that is at rest relative to the frame of Earth, Vega, and Mirror Earth." Quintuplet Q_R is going to travel right ($+x$ – direction) at speed $v = 0.8$, and quintuplet, Q_{MR} is going to travel left ($-x$-direction) at the same speed. *That means Q_{MR}'s velocity is $-v$.* The other three quintuplets will stay where they were born.

We have to choose the origins of the five frames. Probably the most logical origins are for each quintuplet to measure time and space from their own birth. Note that you can only mechanically use L-10 and L-11 when the coordinate origins coincide. Q_E and Q_R have origins that coincide, as do Q_{ME} and Q_{MR}. There is no other commonality. *In addition to using L-10 and L-11 when you can, the length contraction formula and the invariance of the interval are good ways to get results.* It happens that we have five important events as well as the five quintuplets, so we are going to have five tables with five rows each.

Part I. Capture your analysis in tables by filling in all the entries that I haven't filled in. Answer the questions symbolically (in terms of D, γ, and v). Double-check the values I have filled in. It is not impossible for me to mess up on a problem with so many frames and events.

Event 1: Q_E and Q_R are born on Earth

In[∘]:=
```
TableForm[{{0, 0}, {0, 0}, {0, -D}, {0, -2 D}, {"?", "?"}}, TableHeadings →
  {{"E frame", "R frame", "V frame", "ME frame", "MR frame"}, {time, position}}]
```

Out[∘]//TableForm=

	time	position
E frame	0	0
R frame	0	0
V frame	0	−D
ME frame	0	−2 D
MR frame	?	?

Event 2: Q_{ME} and Q_{MR} are born at Mirror Earth

In[∘]:=
```
TableForm[{{0, 2 D}, {"?", "?"}, {0, D}, {0, 0}, {0, 0}}, TableHeadings →
  {{"E frame", "R frame", "V frame", "ME frame", "MR frame"}, {time, position}}]
```

Out[∘]//TableForm=

	time	position
E frame	0	2 D
R frame	?	?
V frame	0	D
ME frame	0	0
MR frame	0	0

Event 3: Q_R and Q_{MR}, going opposite directions, pass Q_V at Vega

In[∘]:=
```
TableForm[{{D / v, D }, {"?", 0}, {D / v, 0}, {D / v, -D}, {"?", 0}}, TableHeadings →
  {{"E frame", "R frame", "V frame", "ME frame", "MR frame"}, {time, position}}]
```

Out[∘]//TableForm=

	time	position
E frame	$\frac{D}{v}$	D
R frame	?	0
V frame	$\frac{D}{v}$	0
ME frame	$\frac{D}{v}$	−D
MR frame	?	0

How much older is Q_V than Q_R and Q_{MR}?

Event 4: Q_{MR} arrives at Earth and meets Q_E

In[∘]:=
```
TableForm[
 {{2 D / v, 0 }, {"?", "?"}, {2 D / v, -D}, {2 D / v, -2 D}, {"?", 0}}, TableHeadings →
  {{"E frame", "R frame", "V frame", "ME frame", "MR frame"}, {time, position}}]
```

Out[∘]//TableForm=

	time	position
E frame	$\frac{2D}{v}$	0
R frame	?	?
V frame	$\frac{2D}{v}$	$-D$
ME frame	$\frac{2D}{v}$	$-2D$
MR frame	?	0

How much older is Q_E than Q_{MR}?

Event 5: Q_R arrives at Mirror Earth and meets Q_{ME}

In[∘]:=
```
TableForm[
 {{2 D / v, 2 D }, {"?", "?"}, {2 D / v, D}, {2 D, 0}, {"?", "?"}}, TableHeadings →
  {{"E frame", "R frame", "V frame", "ME frame", "MR frame"}, {time, position}}]
```

Out[∘]//TableForm=

	time	position
E frame	$\frac{2D}{v}$	$2D$
R frame	?	?
V frame	$\frac{2D}{v}$	D
ME frame	$2D$	0
MR frame	?	?

How much older is Q_{ME} than Q_R?

Part II: Redo all the tables with the numerical values given at the beginning of the problem. You are going to need γ a lot, so first compute that:

$\gamma =$

Event 1: Q_E and Q_R are born at Earth

In[∘]:=
```
TableForm[{{"0 ly", "0 ly"}, {"0 ly", "0 ly"},
  {"0 ly", "-25 ly"}, {"0 ly", "-50 ly"}, {"?", "?"}}, TableHeadings →
  {{"E frame", "R frame", "V frame", "ME frame", "MR frame"}, {time, position}}]
```

Out[∘]//TableForm=

	time	position
E frame	0 ly	0 ly
R frame	0 ly	0 ly
V frame	0 ly	-25 ly
ME frame	0 ly	-50 ly
MR frame	?	?