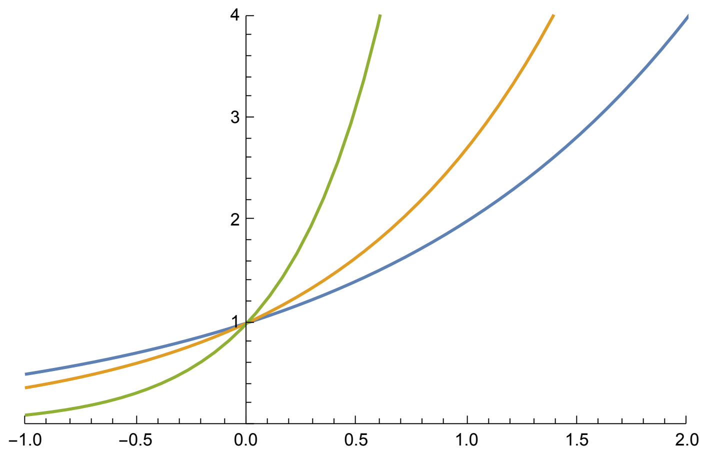
A Primer on Various Exponentials

For Understanding RC Circuits

Exponential Growth for Various Bases — 2, *e*, and 10

In[10]:=
```
Plot[{2^t, E^t, 10^t,}, {t, -1, 2}, PlotRange → {{-1, 2}, {0, 4}}]
```

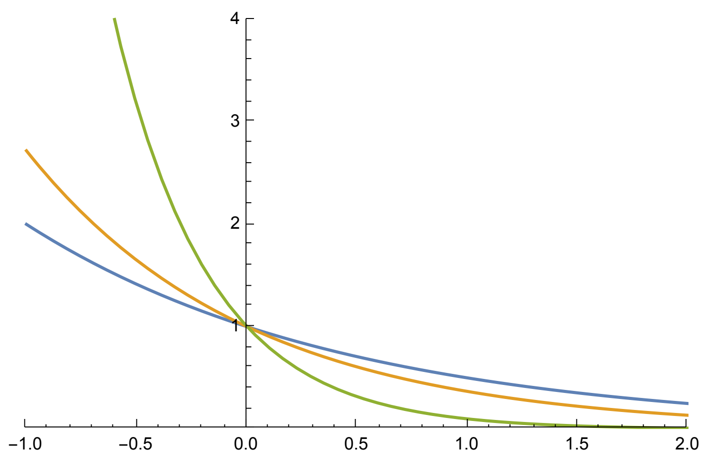
Out[10]=



Exponential Decay for Various Bases — 2, *e*, and 10

In[11]:=
```
Plot[{2^-t, E^-t, 10^-t,}, {t, -1, 2}, PlotRange → {{-1, 2}, {0, 4}}]
```

Out[11]=



A Defining Property of the Exponential

As you can see from the above plots, an exponential with base 10 (green lines) grows and decays more rapidly than an exponential with base *e*, whereas an exponential with base 2 (blue lines) grows and decays less rapidly. This isn't surprising if you know that the numerical value of e is between 2 and 10 (*e* is about 2.71828). But where did this numerical value come from?

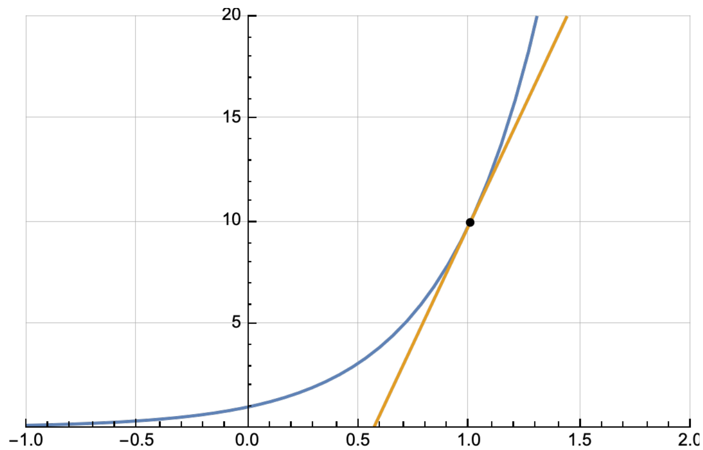
All of these functions have the property that their growth rate (aka "slope") is proportional to their value. This property of exponentials is so fundamental, it could be considered to be defining.

What distinguishes base 2, base e, and base 10 exponentials are their differing constants of proportionality.

Estimate the constant of proportionality for 10^t using the graph below. To improve your estimation, I have drawn the tangent line to 10^t at the point (1, 10).

In[35]:=
```
Plot[{10^t, 10 Log[10] (t - 1) + 10}, {t, -1, 2}, PlotRange → {{-1, 2}, {0, 20}},
 GridLines → Automatic, Epilog → {PointSize[Medium], Point[{1, 10}]}]
```

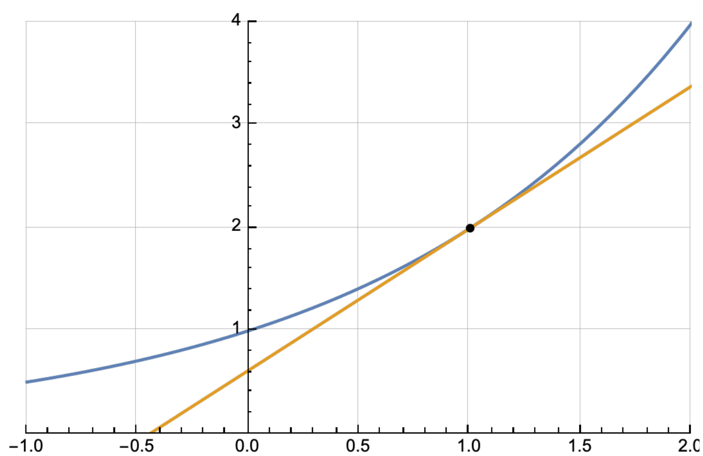
Out[35]=



Estimate the constant of proportionality for 2^t using the graph below. To improve your estimation, I have drawn the tangent line to 2^t at the point (1, 2).

In[36]:=
```
Plot[{2^t, 2 Log[2] (t - 1) + 2}, {t, -1, 2}, PlotRange → {{-1, 2}, {0, 4}},
 GridLines → Automatic, Epilog → {PointSize[Medium], Point[{1, 2}]}]
```

Out[36]=

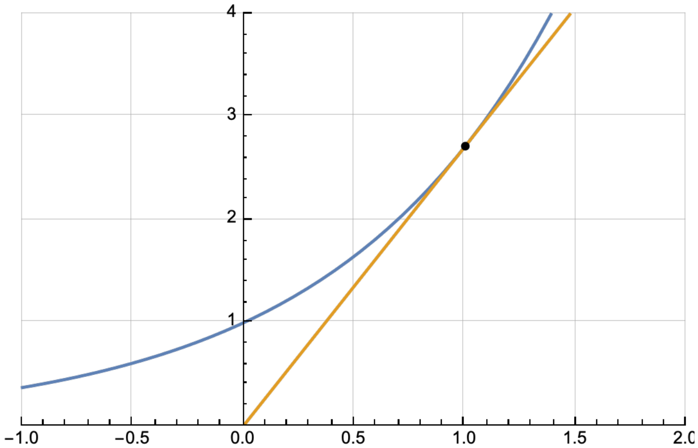


What Makes Base *e* Special and Convenient?

Estimate the constant of proportionality for e^t using the graph below. To improve your estimation, I have drawn the tangent line to e^t at the point $(1, e)$. You don't really need to know (as mentioned above), that the numerical value of e is about 2.71828, but if it helps you make your estimation more concrete, you can use that.

In[37]:=
```
Plot[{E^t, E (t - 1) + E}, {t, -1, 2}, PlotRange → {{-1, 2}, {0, 4}},
 GridLines → Automatic, Epilog → {PointSize[Medium], Point[{1, E}]}]
```

Out[37]=



Repeat, but this time using computing the constant of proportionality by using the tangent that has been drawn at $t = 0$.

In[38]:=
```
Plot[{E^t, t + 1}, {t, -1, 2}, PlotRange → {{-1, 1}, {0, 2}}, GridLines → Automatic,
 AspectRatio → 1, Epilog → {PointSize[Medium], Point[{0, 1}]}]
```

Out[38]=