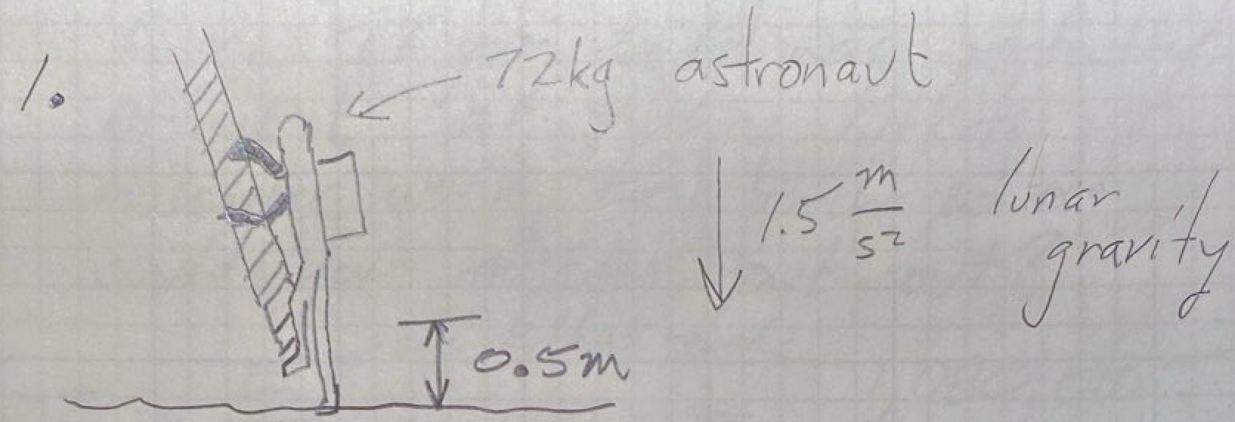
SEPT. 14

# Problem Set 1 – with solutions

1.




Energy released is $72\,kg \cdot 0.5\,m \cdot 1.5\frac{m^2}{s^2}$

That's $48\,kg\frac{m^2}{s^2}$ $\qquad 1\,kg\frac{m^2}{s^2} \equiv 1J$

So the energy released is 48 J.

2. A 50g "serving" of KitKat wafer bars has 259 $\underbrace{\text{kcal}}_{\text{the unit nutritionists use}}$ of energy.

That's 259,000 $\overset{\text{the unit chemists used in the 1800s}}{\text{cal}}$ of energy

a. 1 cal ≡ 4.184 J. How many Joules are in the 50g serving?

$$259{,}000\ \cancel{cal} \cdot \frac{4.184J}{\cancel{cal}} = 1{,}080{,}000\ J$$

I rounded to 3 significant figures

2b. You are hiking, and you have one of those 50 g "servings" every hour. If your body was perfectly efficient at turning KitKats into muscular work, how much power could your muscles put out?

$$\mathcal{P} \equiv \frac{E}{t} = \frac{1{,}080{,}000\text{ J}}{3600\text{ s}}$$

one KitKat serving

every hour

$$= 300\,\frac{\text{J}}{\text{s}} \qquad 1\text{W} \equiv \frac{1\text{J}}{1\text{s}}$$

$$= 300\text{ W}$$

3a. 5 Coulombs of charge "falls" through an electrical potential and releases 150 J of energy. What is the electrical potential?

$$\mathcal{V} \equiv \frac{E}{Q} = \frac{150\text{ J}}{5\text{ C}} = 30\,\frac{\text{J}}{\text{C}}$$

$$1\,\frac{\text{J}}{\text{C}} \equiv 1\text{V}$$

$$\mathcal{V} = 30\text{V}$$

I started using script $\mathcal{V}$ for voltage (aka electrical potential) so that it is distinguished from Roman V for Volts.

3b. 20A is $20\frac{C}{s}$

20C "falling" through 12V is 240J. That is happening every second. So the power is $P = \frac{E}{t} = \frac{240J}{s} = 240W$

4a. A proton has charge of $1.6 \times 10^{-19}C$.

<u>I'm splitting the question into two parts.</u>

How many protons make 1C?

$$\frac{1\cancel{C}}{1.6 \times 10^{-19}\frac{\cancel{C}}{\text{proton}}} = \frac{1}{1.6} \times 10^{19}\text{protons}$$

$$\frac{1}{1.6} = \frac{1}{8/5} = \frac{5}{8} = 0.625$$

$$= 0.625 \times 10^{19}\text{protons}$$

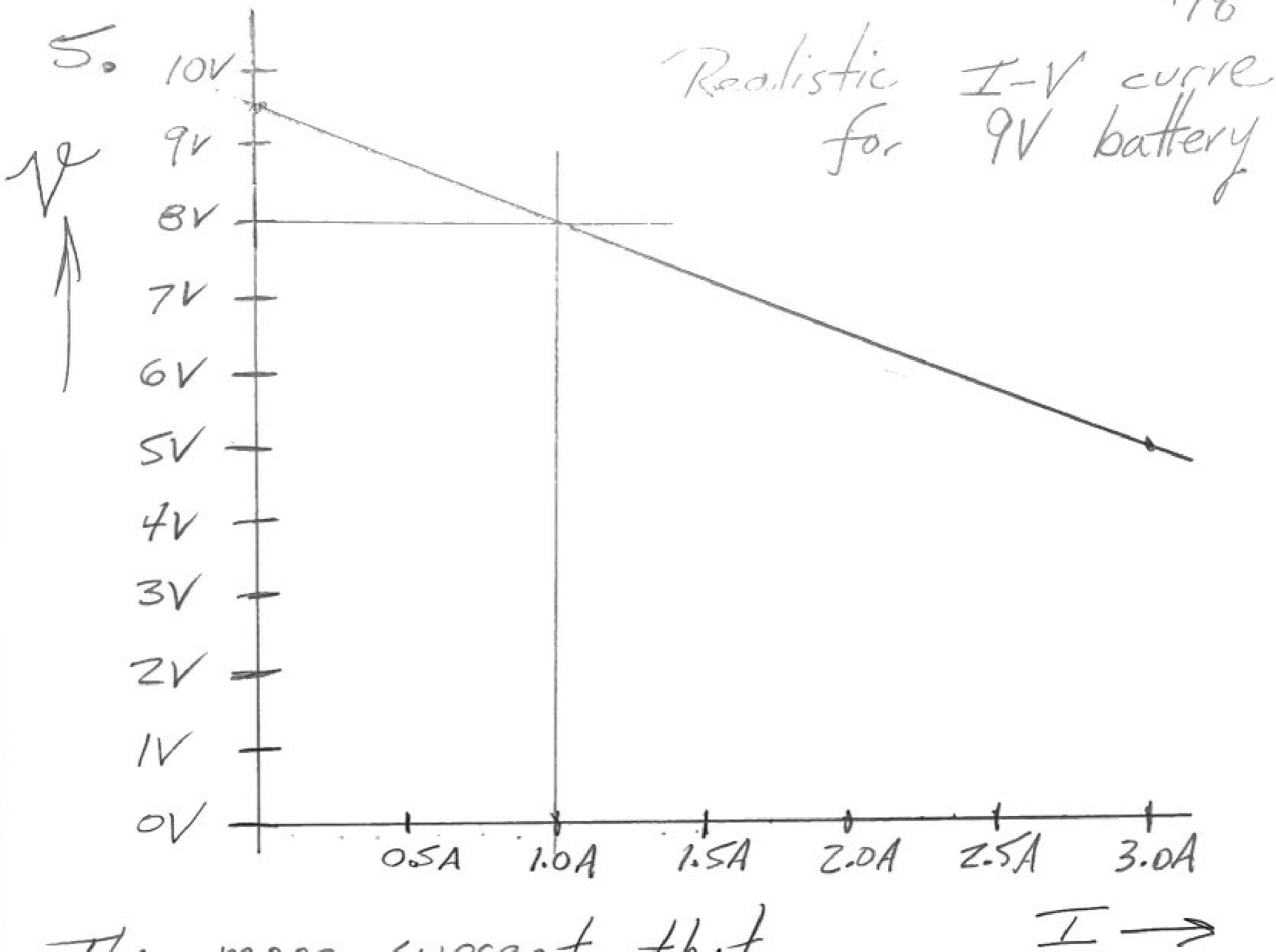
$$= 6.25 \times 10^{18}\text{protons}$$

4b. If the LHC at CERN carries 0.58A in its beam, how many protons pass by the particle detectors in 1 second?

$0.58A \times 1s = 0.58C$

$$0.58\cancel{C} \times 6.25 \times 10^{18}\frac{\text{protons}}{\cancel{C}} = 3.6 \times 10^{18}\text{protons}$$

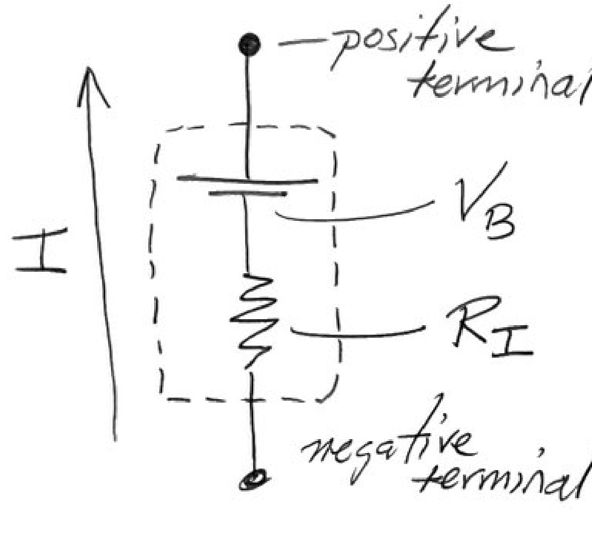
5.




The more current that is drawn from the battery, the less voltage it delivers.

A realistic battery can be modeled as a perfect voltage source and a resistor.

What is in the dotted lines models the realistic battery



5a. From the graph, determine $V_B$.

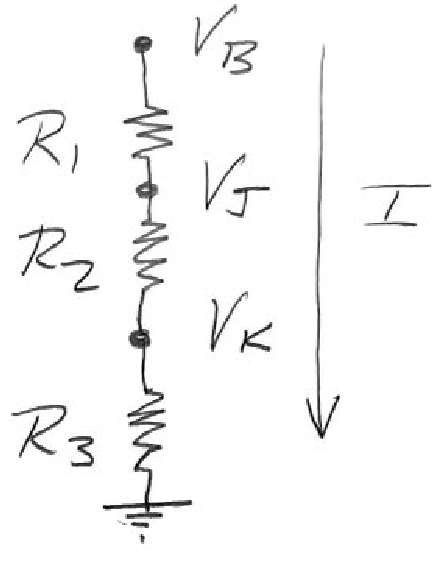
When no current is flowing we read 9.5V off the graph. So $V_B = 9.5V$

5b. From the graph, determine $R_I$.

When 1A is flowing, the battery's voltage has dropped to 8V. That is a drop of 1.5V relative to when no current is flowing. The resistor must be responsible for that 1.5V drop in voltage.

$$R_I = \frac{1.5V}{1A} = 1.5\,\Omega$$

6a. (This was mis-numbered as 5a.)




Determine $V_J$ and $V_K$ in terms of $V_B$, $R_1$, $R_2$, and $R_3$. $I$ is also an unknown that shouldn't appear in your answers.

6a. (CONT'D) Voltage across $R_1$ is $V_B - V_J$

" " $R_2$ is $V_J - V_K$

" " $R_3$ is $V_K - 0$

Ohm's Law applied to each resistor:

$$IR_1 = V_B - V_J$$

$$IR_2 = V_J - V_K \qquad (*)$$

$$IR_3 = V_K - 0 \qquad (**)$$

Quick way to find $I$ so you can eliminate it: add all three equations.

$$IR_1 + IR_2 + IR_3 = V_B - V_J + V_J - V_K + V_K - 0$$

$$I(R_1 + R_2 + R_3) = V_B$$

$$I = \frac{V_B}{R_1 + R_2 + R_3}$$

Now use the $(**)$ equation to find $V_K$.

$$V_K = IR_3 = V_B \frac{R_3}{R_1 + R_2 + R_3}$$

Then use the $(*)$ equation to find $V_J$.

I skipped a couple of steps

$$V_J = V_K + IR_2 = V_B \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

6b. (This was mis-numbered as 5b.)

Pluge in $V_B = 10V$ $R_1 = 800\Omega$ and $R_2 = R_3 = 100\Omega$

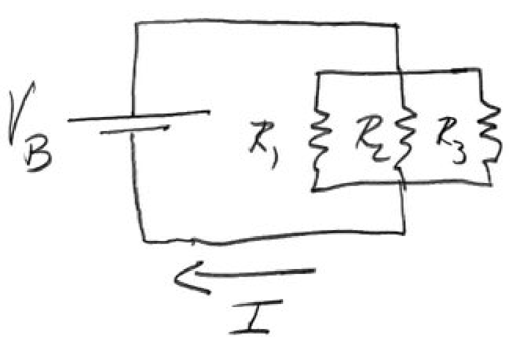
$$V_J = 10V \frac{100\Omega + 100\Omega}{800\Omega + 100\Omega + 100\Omega}$$

$$= 10V \frac{200\Omega}{1000\Omega} = 10V \cdot \frac{2}{10} = 2V$$

$$V_K = 10V \frac{100\Omega}{1000\Omega} = 10V \cdot \frac{1}{10} = 1V$$

Can you see why this is called a voltage divider?

7a. (This was mis-numbered as 6a.)




Find $I$ in this circuit.

Each resistor has a voltage $V_B$ applied to it. Therefore

$$I_1 = \frac{V_B}{R_1} \quad I_2 = \frac{V_B}{R_2} \quad I_3 = \frac{V_B}{R_3}$$

7a. (CONT'D) The total current

$$I = I_1 + I_2 + I_3 = \frac{V_B}{R_1} + \frac{V_B}{R_2} + \frac{V_B}{R_3}$$

$$= V_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

7b. Rearrange the equation to solve for $\frac{V_B}{I}$

$$\frac{V_B}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

This shows that the three resistors in parallel act as if they are a single resistor whose resistance is: $R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

This is more commonly written as:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$